Dokazi bez riječi

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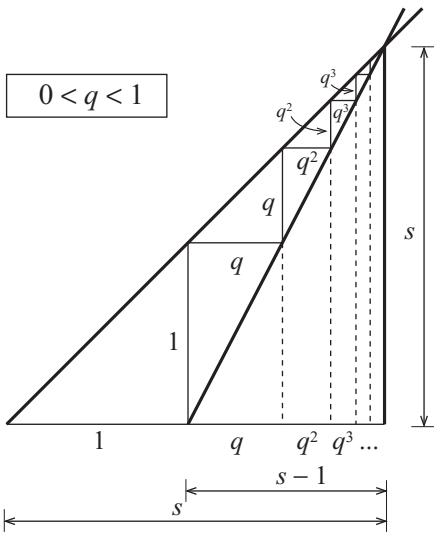
Geometrijski red bez riječi

1. dokaz

$$
\begin{array}{l}
1 \quad : (1-q) = 1 + q + q^2 + q^3 + \ldots \\
\underline{-(1-q)} \\
\quad q \\
\quad \underline{-(q-q^2)} \\
\qquad q^2 \\
\qquad \underline{-(q^2-q^3)} \\
\qquad\quad q^3 \\
\qquad\quad \ddots \\
\qquad\qquad \Rightarrow \ 0 \quad \text{za} \quad |q| < 1.
\end{array}
$$

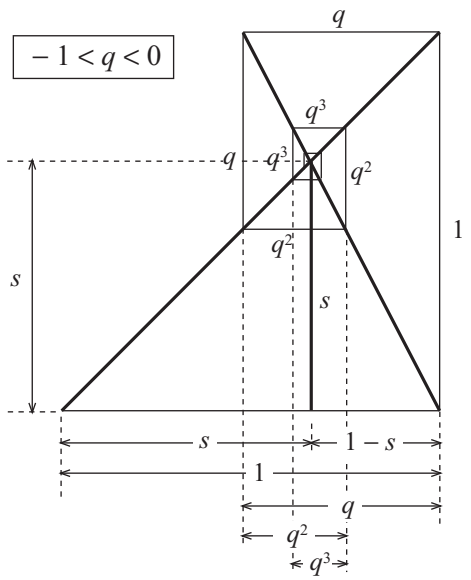
2. dokaz



$$\frac{s}{s-1} = \frac{1}{q}$$

$$\Downarrow$$

$$s = \frac{1}{1-q}$$

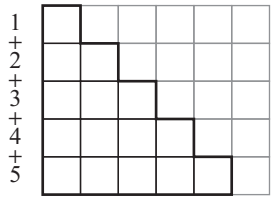


$$\frac{s}{s-1} = \frac{1}{q}$$

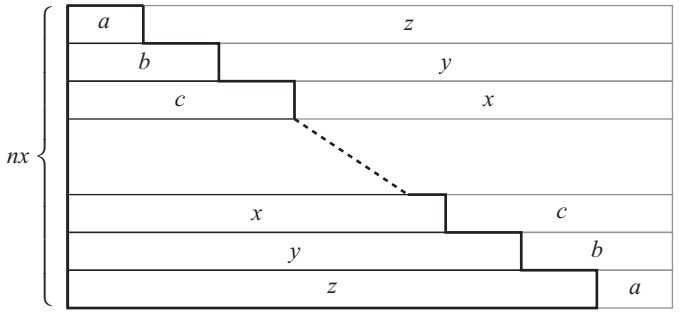
$$\Downarrow$$

$$s = \frac{1}{1-q}$$

Aritmetički red bez riječi



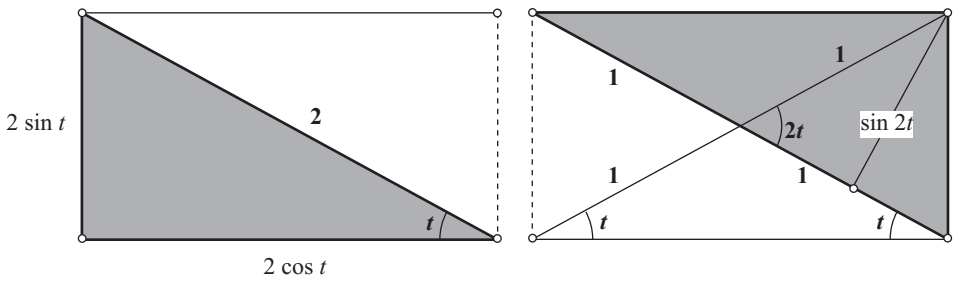
$$1 + 2 + \ldots + n = \frac{1}{q}$$



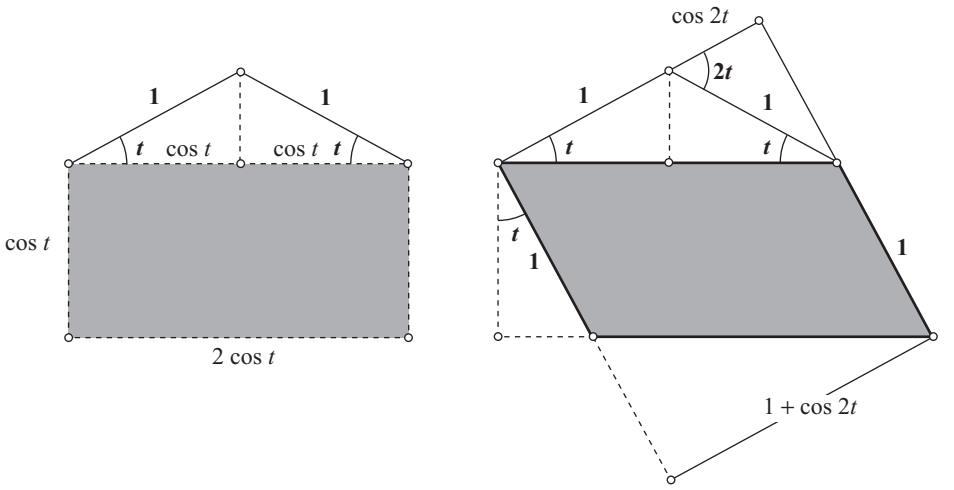
$$a + b + c + \ldots + x + y + z = \frac{n \cdot (a+z)}{2}$$

Trigonomerijski dokazi bez riječi

1. Formule dvostrukih kutova



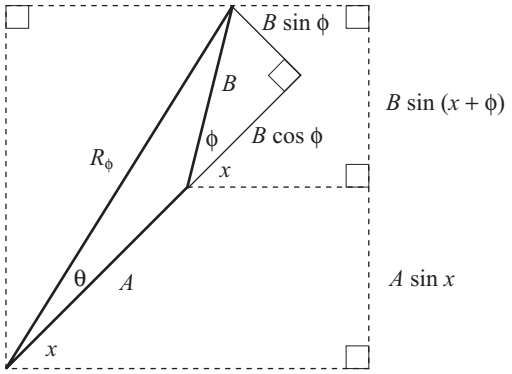
$$2 \sin t \cos t = \sin 2t$$



$$2 \cos^2 t = 1 + \cos 2t$$

(YIHNAN DAVID GAU, CALIFORNIA STATE UNIVERSITY)

2. Zbrajanje poput sinusa



$$R_\phi = \sqrt{A^2 + B^2 + 2AB\cos\phi} \qquad \tan\theta = \frac{B\sin\phi}{A + B\cos\phi}$$

$$A\sin x + B\sin(x+\phi) = R_\phi \sin(x+\theta)$$

$$\phi = \pi/2 \;\Rightarrow\; \tan\theta = \frac{B}{A}$$

$$A\sin x + B\cos x = \sqrt{A^2 + B^2}\,\sin(x+\theta)$$

(RICK MABRY, PAUL DEIERMANN, LUISIANA STATE UNIVERSITY)