THE IRONY OF THE STANDARD PROOF OF $d\sin j \,/\, dj = \cos j$

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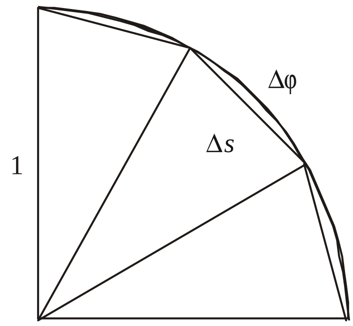
The arclength of the part of the unit circle is defined using polygons with small sides:

$$j = \lim_{\Delta s \to 0} (\Sigma \Delta s)$$



From this definition it follows that

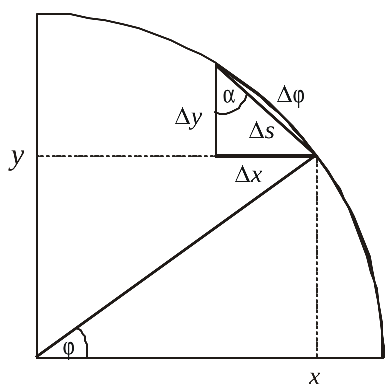
(1) $$\lim_{\Delta j \to 0} \frac{\Delta s}{\Delta j} = 1,$$

If $x = \cos j$ and $y = \sin j$ are defined as coordinates of the point on the unit circle with the corresponding arclength j, then for their derivatives we have:

$$\frac{d}{dj}\sin j = \frac{dy}{dj} = \lim_{\Delta j \to 0} \frac{\Delta y}{\Delta j} = \lim_{\Delta j \to 0} \frac{\Delta y}{\Delta s}\frac{\Delta s}{\Delta j} = \lim_{\Delta j \to 0} \frac{\Delta y}{\Delta s} \lim_{\Delta j \to 0} \frac{\Delta s}{\Delta j} =$$

$$= \text{cf. (1)} = \lim_{\Delta j \to 0} \frac{\Delta y}{\Delta s} = \lim_{\Delta j \to 0} (\cos a) = \cos j .$$

The companion formula $d\cos j \,/\, dj = -\sin j$ can be proved in the same way.



How did we manage to prove $d\sin j \,/\, dj = \cos j$, in such a simple way? Where did the infamous

(2) $$\lim_{\Delta j \to 0} \frac{\sin \Delta j}{\Delta j} = 1$$

disappeared? Of course, it is contained in the definition of the arclenght j i. e. it is contained in (1). (Formulae (1) and (2) are obviously equivalent.) Basically, the standard proof of $d\sin j \,/\, dj = \cos j$ is difficult because it take pains in establishing (2) although it already presupposes (2) in the form (1). Namely, it presupposes it in the very definition of j which is, of course, presupposed in the definition of $\sin j$. Really ironic.