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Applied Logic

60 sati

This is a general background course intended for

- mathematicians interested in the foundations of their discipline and/or computer–assisted deduction;
- computer–scientists looking beyond routine applications;
- potential logicians.

Some familiarity with the contents of undergraduate courses *Mathematical Logic*, and also *Set Theory, Mathematical Theory of Computation*, while not a formal prerequisite, would be helpful: the topics discussed in these courses will not be repeated in detail.

Preliminaries. Induction and recursion. Finite and infinite games. Ramsey's theorem. Syntax and interpretation of logical languages.

Computability. Models of computation, recursive and recursively enumerable sets. Rice's theorem. The decision problem, theorems of Church and Trakhtenbrot. Reductions, completeness and the arithmetical hierarchy.

Computational complexity. Time and space, complexity classes, reductions, completeness. The classes LOGSPACE, P, NP, coNP, PSPACE, EXPTIME. Logical examples of complete problems.

Propositional logic. Semantics, normal forms, completeness, compactness, interpolation, definability.

First order quantificational logic. Semantics, normal forms, completeness, compactness, interpolation, definability. Nonstandard models. Expressivity. Cannonical models, the theorem of Herbrand, resolution and logic programming: a way towards computational deduction.

First order arithmetics. Proofs, algorithms and reflexion. Limitative theorems of Tarski, Gödel and Rosser.

Introduction to model theory. Models and their morphisms. Classes and constructions of models, closure properties. Ehrenfeucht–Fraïssé games. Quantifier elimination and some decidable classes. Finite models and 0–1 laws. The scope of first order and the theorem of Lindstöm.

Beyond first order. Expressivity of 2. order: how to formalize Peano and Dedekind. Between 1. and 2. order: formal languages and Büchi machines, fixed–point logics, Fagin's theorem and descriptive complexity.

Introduction to proof–theory. Natural deduction and the Curry–Howard isomorphism. Normalization and functional programming with proofs. Tactical computational deduction (HOL, PVS, Isabelle, Coq, ...). Sequent calculi, cut–elimination and validity tests (tableau–methods).

Nonclassical logics. Intuitionistic, modal and linear logics. Logics of actions and programs.

Essential references

R. Cori, D. Lascar, *Mathematical Logic, Part 1*, Oxford University Press 2000.

R. Cori, D. Lascar, *Mathematical Logic, Part 2*, Oxford University Press 2001.

M. Vuković, *Matematička logika 1*, PMF MO, Zagreb 2000.

Recommended references

J. Bell, A. Machover, *A Course in Mathematical Logic*, North–Holland 1977.

E. Börger, E. Grädel, Y. Gurevich, *The Classical Decision Problem*, Springer 1997.

K. Doets, *Basic Model Theory*, CSLI Publications 1996.

H. Ebbinghaus, J. Flum, *Finite Model Theory*, Springer 1999.

H. Schwichtenberg, A. Troelstra, *Basic Proof Theory*, Cambridge University Press 2000.