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SHIP RAMP MOTION ANALYSIS ON WAVES IN SHALLOW WATER

Summary

The motion analysis of moored ships is very important factor to consider the berth efficiency during year. The solution methodology consists of modelling a ship as panel model used to calculate the hydrodynamic loads and responses from potential theory. The mooring lines are modelled by ship to ground spring elements. The stiffness of those elements is accumulated into the global restoring matrix for the rigid body equation of motion. The system of obtained differential equations is solved by frequency domain procedure taking explicitly into account the influence of shallow water. The effect of irregular waves is taken into account by appropriate wave spectrum. Illustrative applications of the method are given for a two ferryboats and one cruiser moored on the outside of hypothetic jetty at Adriatic Sea harbour. The irregular sea is described by the Tabain spectrum. The results are presented as comparison of significant ship ramp motion amplitudes and chosen specific criteria limit.

Key words: *ship mooring, ship motion, terminal operability*

ANALIZA POMAKA RAMPE BRODA NA VALOVLJU U PLIĆINI

Sažetak

Analiza njihanja vezanog broda veoma je značajna sa stanovišta operativnosti pojedinog privezišta tijekom godine ili sezone. Metodologija rješenja sastoji se u modeliranju broda panel metodom u cilju proračuna hidrodinamičkih opterećenja i odziva primjenom potencijalne teorije. Privezne linije modelirane su kao posebni opružni elementi i njihov je utjecaj na gibanje uključen preko matrice povratnih koeficijenata. Sustav dobivenih diferencijalnih jednadžbi riješen je u frekvencijskoj domeni uzimajući u obzir utjecaj dna. Utjecaj nepravilnih valova uzet je u obzir pomoću odgovarajućeg spektra valova. Prikaz primjene postupka dat je na primjeru dva trajekta i jednog broda za krstarenje vezanih na hipotetskom vanjskom vezu jadranske luke. More je opisano Tabainovim spektrom valova. Rezultati su prikazani kao usporedba značajnih vrijednosti amplituda gibanja rampe broda i graničnih vrijednosti zadanih kriterija.

Ključne riječi: *privez broda, njihanje broda, operativnost privezišta*

1. Introduction

The motion amplitude of ship moored in a harbour is affected by not only sea waves but also by the mooring arrangements. Because of exaggerated ship ramp motion these motions can affect the possibility of loading and unloading and consequently reduce the efficiency of berth. So it is important to make a reliability assessment of number of non-operative days during the year or season. For the purpose of transfer function calculation it is necessary to make a reliable model of ship with mooring arrangement that must satisfy equilibrium equation and compatibility relations. The irregular waves can be described by appropriate sea spectrum. If the water at the berth place is shallow the sea depth must be taken into account. Significant amplitude of absolute horizontal and vertical ramp motion is accepted as the criteria for the moored ship operability.

2. Mathematical background

2.1. Wave loads on ship

The assumption of potential flow allows defining the velocity flow as the gradient of the velocity potential Φ that satisfies the Laplace equation

$$\nabla^2\Phi = 0 \qquad \textbf{(1)}$$

in the fluid domain [1]. The harmonic time dependence allows defining a complex velocity potential ϕ related to Φ by

$$\Phi = \mathrm{Re}\left(\phi\, e^{i\omega t}\right) \qquad \textbf{(2)}$$

where ω is the frequency of the incident wave and t is time. The associated boundary-value problem will be expressed in terms of the complex velocity potential with the understanding that the product of all complex quantities with the factor $e^{i\omega t}$ applies. The liberalized form of the free-surface condition is

$$\frac{\partial\Phi}{\partial z} + \frac{1}{g}\frac{\partial^2\Phi}{\partial t^2} = 0 \qquad \textbf{(3)}$$

where g is the acceleration of gravity. The velocity potential of the incident wave is defined by

$$\phi_0 = \frac{ig\zeta_a}{\omega}\frac{\cosh k(z+d)}{\cosh kd} e^{-k(x\cos\beta + y\sin\beta)} \qquad \textbf{(4)}$$

where ζ_a is the wave amplitude, the wave number k is the real root of the dispersion relation, d is the sea depth and β is the angle between the direction of propagation of the incident wave and the positive x-axis. Linearisation of the problem permits decomposition of the velocity potential ϕ into the radiation ϕ_R and diffraction component ϕ_D:

$$\phi = \phi_D + \phi_R = \phi_0 + \phi_7 + \sum_{j=1}^{6}\xi_j\phi_j \qquad \textbf{(5)}$$

Constants ξ_j denote the complex amplitudes of the body oscillatory motion in its six rigid-body degrees of freedom and ϕ_j the corresponding unit-amplitude radiation potentials. The velocity potential ϕ_7 represents the disturbance of the incident wave by the body fixed at its undisturbed position. The total diffraction potential ϕ_D denotes the sum of ϕ_7 and the incident wave potential.

On the undisturbed position of the body boundary the radiation and diffraction potentials are subject to the conditions

$$\frac{\partial \phi_j}{\partial n} = i\omega n_j \qquad \frac{\partial \phi_D}{\partial n} = 0 \qquad \textbf{(6)}$$

where $(n_1, n_2, n_3) \equiv \mathbf{n}$ i $(n_4, n_5, n_6) \equiv \mathbf{r} \times \mathbf{n}$ and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. The unit vector **n** is normal to the body boundary and points out of the fluid domain. The boundary value problem must be supplemented by a condition of outgoing waves applied to the velocity potentials ϕ_j, j=1,...,6. Boundary defined in this way assumes ship anchored in open sea. Influence of berth presence on velocity potential is neglected.

The submerged half part of the ships under considerations is modelled with 3D panels using SESAM Software Package [2] as presented on Figure 1. The radiation and diffraction velocity potentials on the wet part of the body surface are determined from the solution of an integral equation obtained by using Green's theorem with the free surface source potentials as the Green's functions. The source strengths are evaluated based on the source distribution method using the same source potentials. The integral equation is discretised into a set of algebraic equations by approximating the body surface with a number of plane quadrilateral panels. The source strengths are assumed to be constant over each panel. One plane of symmetry of the body geometry is present. The solution of the algebraic equation system provides the strength of the sources on the panels. The equation system, which is complex and indefinite, is solved by an iterative method. Also possible contact between ship and berth are neglected.

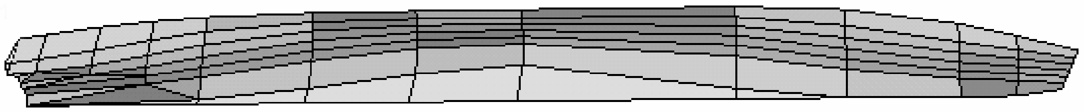

Fig. 1 3D panel model of ferryboat

Slika 1. 3D panel model trajekta

2.2. Mooring modelling

The mooring lines (Figure 2) are assumed to be weightless and with linear stiffness characteristics. The external restoring forces from mooring lines in ship motion model are included by mooring elements. The mooring elements are defined at appropriate nodes on the ship model. The hydro properties of a mooring element include the element orientation, the pre-tension and the restoring characteristics [3, 4]. The restoring contributions from the mooring elements are assembled into the body restoring matrix and hence contribute to the rigid body motion. The rigid body motion computed yields dynamic restoring forces acting in the mooring element nodes.

The mooring stiffness matrices $\mathbf{K}_m$ for each mooring element in a model are described by spring constants and the pre-tension in x, y, and z direction. The $\mathbf{K}_m$ matrices are accumulated into the global restoring matrix for the rigid body equation of motion. Since the $\mathbf{K}_m$ matrices are established directly in the motion reference coordinate system, no transformations are needed in the accumulation process. Having solved the equation of motion $\mathbf{x}_g$ represents the global motion of the rigid body system. The force vector $\mathbf{f}_g$ for each fairlead node, described in the result reference coordinate system, is then computed as

$$\mathbf{f}_g = \mathbf{K}_m \mathbf{x}_g \qquad \textbf{(7)}$$

2.3. Global motion responses

The equation of motion in established for harmonic motion of rigid body systems expressed in the global coordinate system (Figure 2). By applying Newtons law and including the added mass, damping and exciting force contributions acting on the panel and parts of a mooring hydro model the complex motion vector $\mathbf{H}(\omega,\beta) = (\eta_1, \eta_2..., \eta_6)$ can be found from the equation of motion

$$\left[\ -\omega^2\left(\mathbf{M}+\mathbf{A}(\omega)\right)+i\omega\,\mathbf{B}_p(\omega)+\mathbf{B}_v+\mathbf{C}+\mathbf{C}_e\ \right]\mathbf{H}(\omega,\beta)=\mathbf{F}(\omega,\beta) \tag{8}$$

where **M** represents the body inertia matrix, $\mathbf{A}(\omega)$ represents the frequency dependent added mass matrix, $\mathbf{B}(\omega)_p$ represents the frequency dependent potential damping matrix, $\mathbf{B}_v$ represents the linearised viscous damping matrix, **C** represents the hydrostatic restoring matrix, $\mathbf{C}_e$ represents the external restoring matrix and $\mathbf{F}(\omega,\beta)$ is the complex exciting force vector for frequency ω and incident wave heading angle β.

The eigenvalues λ and eigenvectors $\mathbf{\Psi}$ of the rigid body system is obtained for a given incident wave frequency by solving the eigenvalue problem

$$\left[-\lambda\left(\mathbf{M}+\mathbf{A}(\omega)\right)+\mathbf{C}\right]\mathbf{\Psi}=0 \tag{9}$$

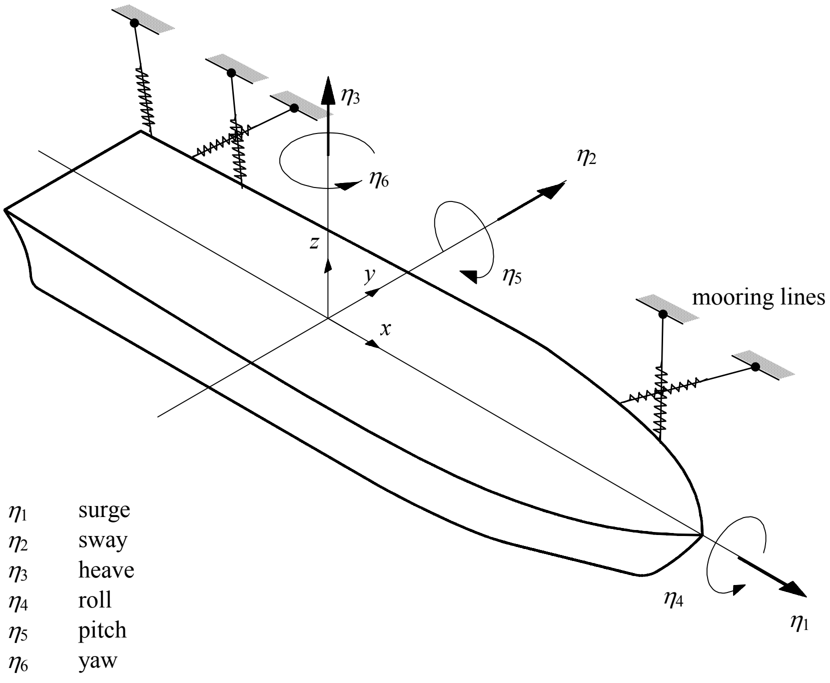

Fig. 2 Global coordinate system

Slika 2. Globalni koordinatni sustav

2.4. Environmental description

The waves are described by the Airy wave theory. The incident waves may be specified by either wave lengths, wave angular frequencies or wave periods. The direction of the incident waves are specified by the heading angle β between the positive x-axis and the propagating direction. The incident wave ζ is defined as

$$\zeta = \mathrm{Re}\left[\ \zeta_a e^{i\left(\omega t-k\left(x\cos\beta+y\sin\beta\right)\right)}\ \right] \tag{10}$$

The finite depth dispersion relation used in the above expressions is $\omega^2 = gk\tanh\left(kd\right)$.

The sea are described by Tabain wave spectrum [5]:

$$S_{\zeta}(\omega) = 0{,}862 \frac{0.0135 g^2}{\omega^5} \exp\left(-\frac{5.186}{\omega^4 H_s} \right) 1.63^{\exp\left(-\frac{(\omega-\omega_m)^2}{2\sigma^2 \omega_m^2} \right)} \tag{11}$$

for the range of significant wave height H_s, where the modal frequency ω_m is estimated by $\omega_m = 0.32 + \frac{1.80}{H_s + 0.60}$ and parameter σ is calculated as:

$$\begin{aligned} \sigma &= 0{,}08 \quad \text{za} \quad \omega < \omega_m \\ \sigma &= 0{,}10 \quad \text{za} \quad \omega > \omega_m \end{aligned} \tag{12}$$

2.5. Numerical example

The application of the computational method is given for two ferryboats of approximately 75 m (ship A) and 115m (ship B) lengths and one cruiser of approximately 200 m length (ship C) moored to a pier on the outside of hypothetical Adriatic Sea harbour jetty. The moored ship transfer functions of absolute vertical ramp motion of ship A, B and C, shown in Figures 3 to 5 as a function of wave frequency, have been computed by using software package SESAM [2]. The sea depth for the ship A, B and C are respectively 6.5 m, 10 m and 15 m. It is used in the calculation of Green's functions for finite water depth.

The corresponding response spectrums for absolute vertical ramp motion of ferryboats A, B (bow ramp) and C (side ramp) for the South wind waves are shown in Figures 6 and 7. Figure 8 represents the same for the cruiser C on beam waves that are the most unfavourable situation for the lateral ramp of this ship. The resulting spectrums are then processed to obtain the corresponding order statistics. They constitute the input data for the estimation of criteria limit exceedance. Figures 9, 10 and 11 show respectively significant absolute longitudinal, transversal and vertical motion amplitude for all three ships as a function of sea state defined by significant wave height. The calculation is done for South wind waves that seem to be the highest waves in this area during summer. The limits for significant amplitude of vertical and transversal motion are set to be 0.5 m and limit for significant amplitude of longitudinal motion 0.1 m. These criteria limit has been implemented according to the recommendation of port and ship authorities. From those diagrams and from the sea state statistics for the considered area from May to October it is possible to estimate the limiting significant wave height and the number of non-operative days *NOD* during season. These data and the operability are showed at Table 1 for exposed berths of the port in Adriatic Sea.

Table 1 Operating limits and non-operative days

Tablica 1. Granične vrijednosti operativnosti i broj neoperativnih dana

Ship	From May to October (Jugo)			
	Hs, m	*NOD*, days	Operability, %	Limiting criteria
Ferryboat A	1.25	7	96	Longitudinal motion
Ferryboat B	1.7	1	99	Vertical motion
Cruiser C	1.9	0	100	Vertical motion

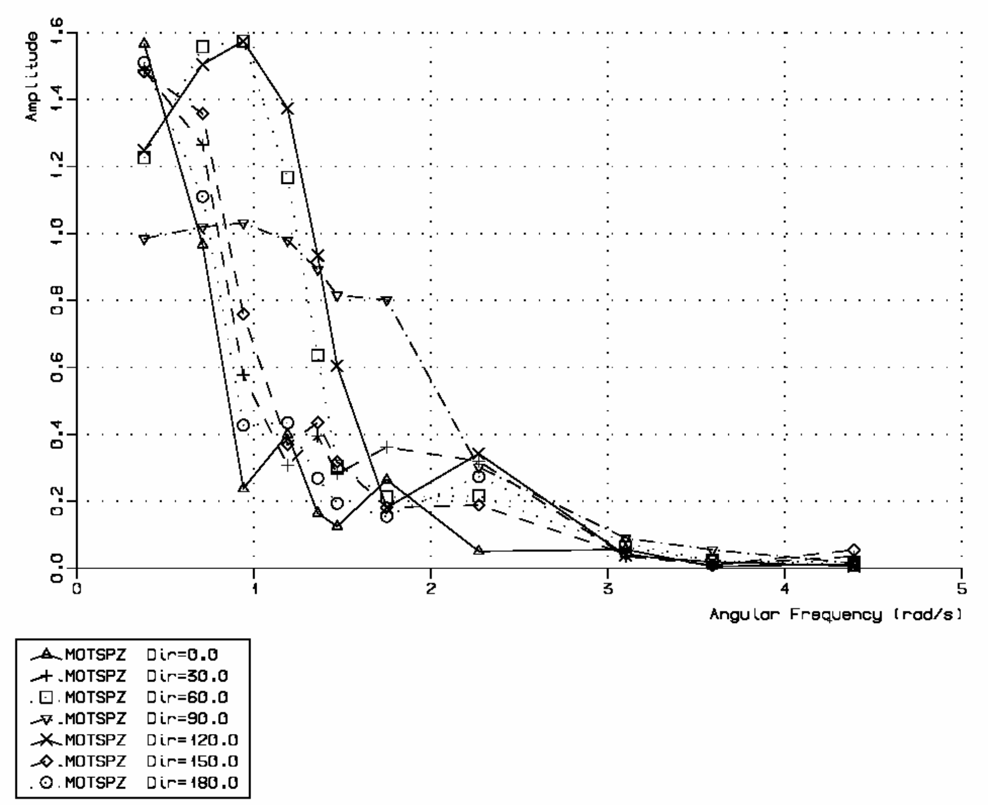

Fig. 3 Vertical motion transfer function of ship A ramp

Slika 3. Vertikalno gibanje rampe broda A

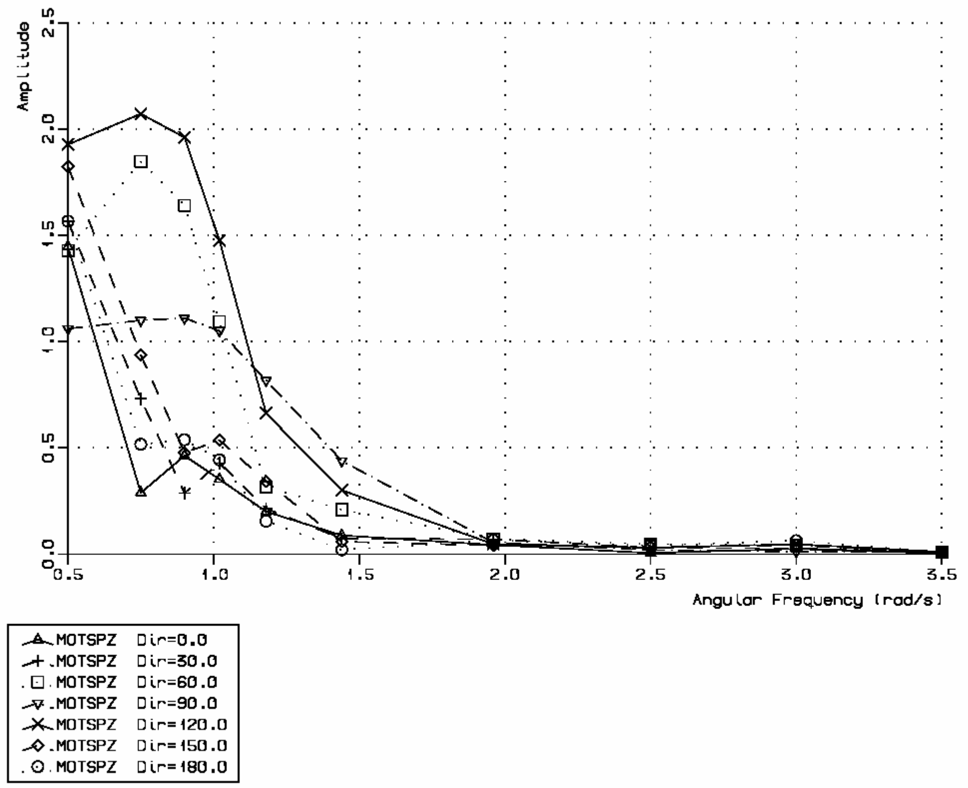

Fig. 4 Vertical motion transfer function of ship B ramp

Slika 4. Vertikalno gibanje rampe broda B

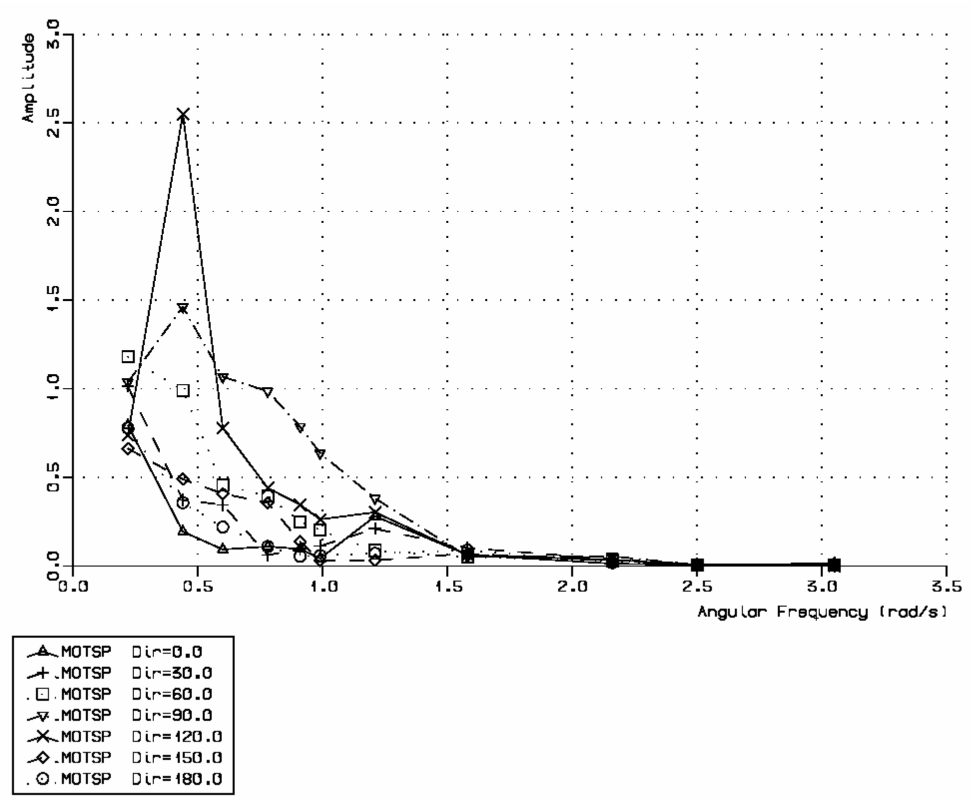

Fig. 5 Vertical motion transfer function of ship C ramp

Slika 5. Vertikalno gibanje rampe broda C

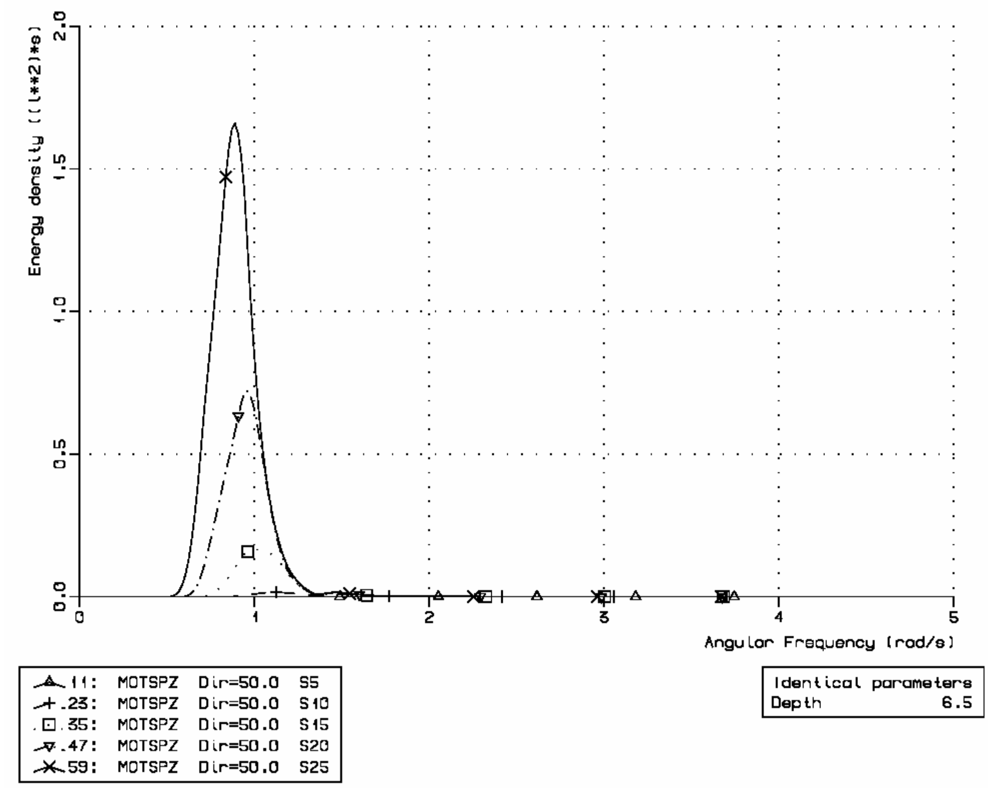

Fig. 6 Response spectrum of ramp vertical motion for South Wind waves for ship A

Slika 6. Spektar vertikalnog gibanja rampe na valovima Juga za brod A

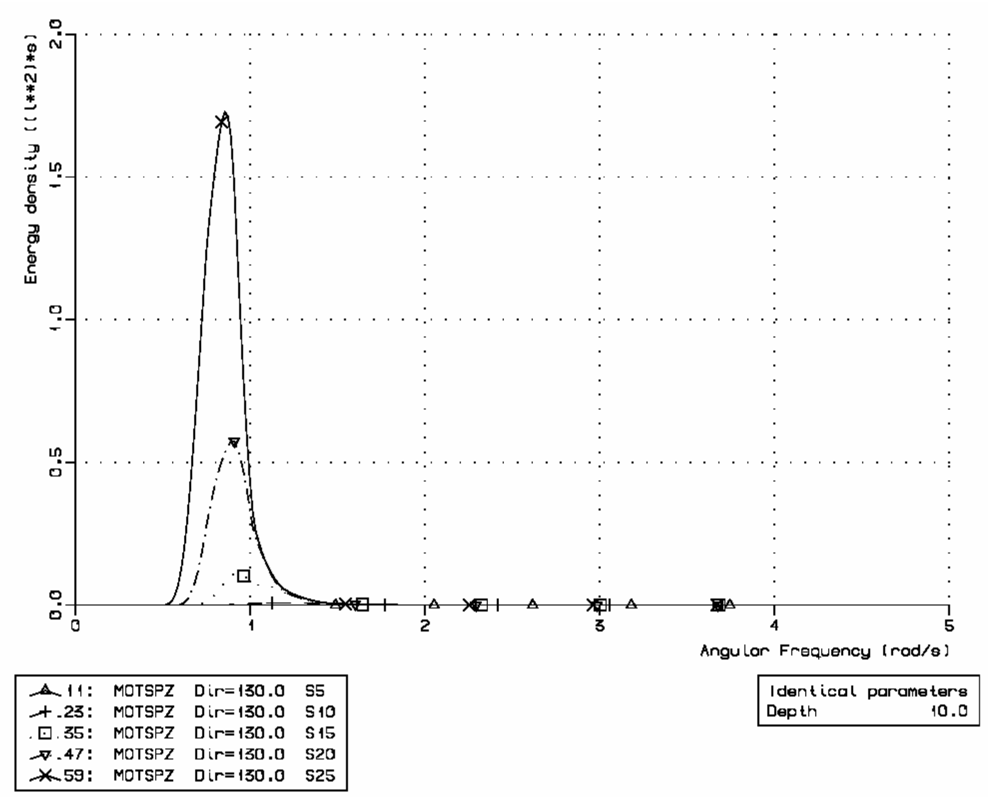

Fig. 7 Response spectrum of ramp vertical motion for South Wind waves for ship B

Slika 7. Spektar vertikalnog gibanja rampe na valovima Juga za brod B

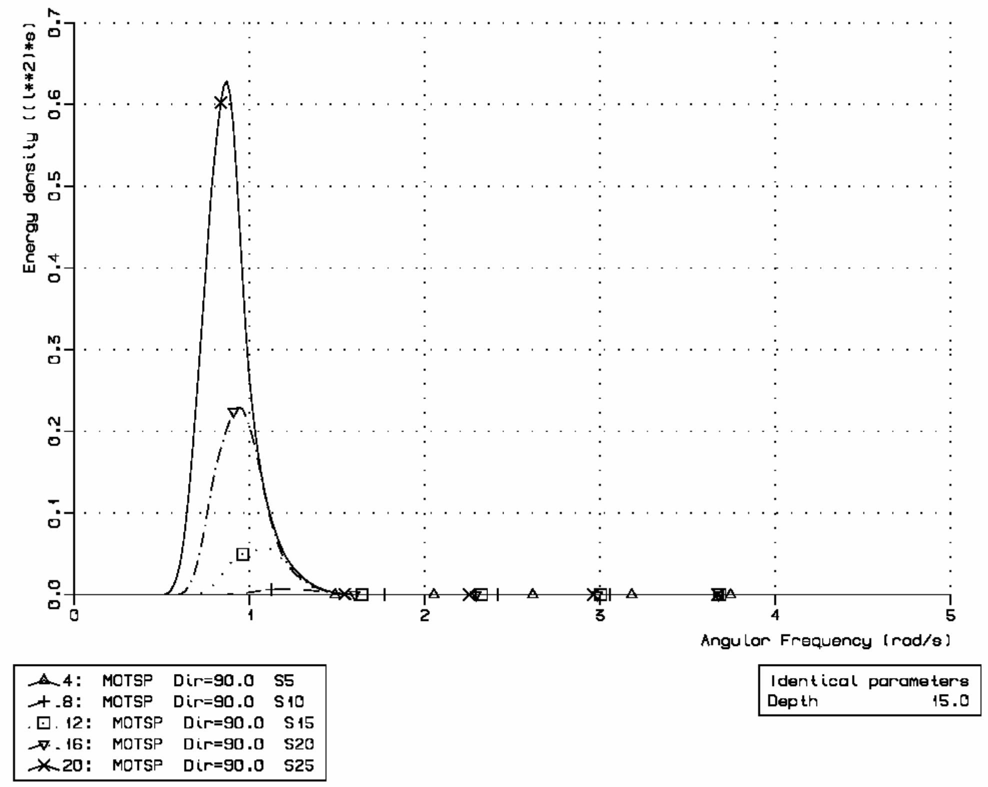

Fig. 8 Response spectrum of ramp vertical motion for beam waves for ship C

Slika 8. Spektar vertikalnog gibanja rampe na bočnim valovima za brod C

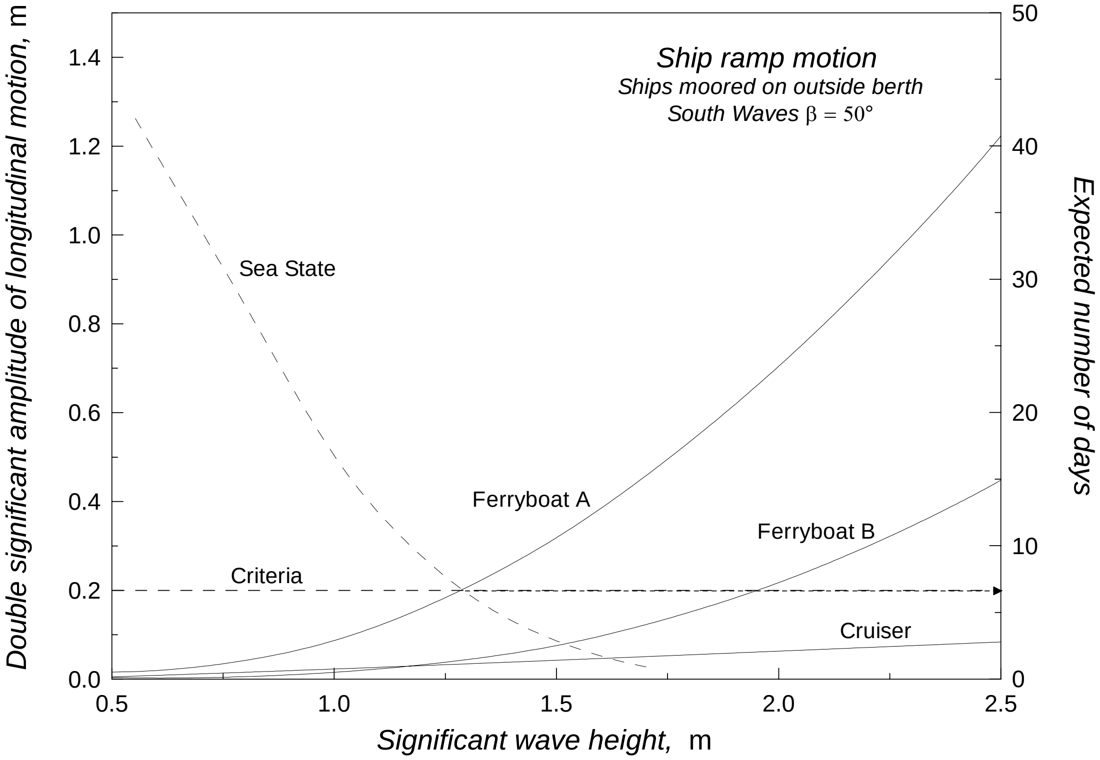

Fig. 9 Significant amplitude of ship ramp longitudinal motion

Slika 9. Značajna amplituda uzdužnog gibanja rampe

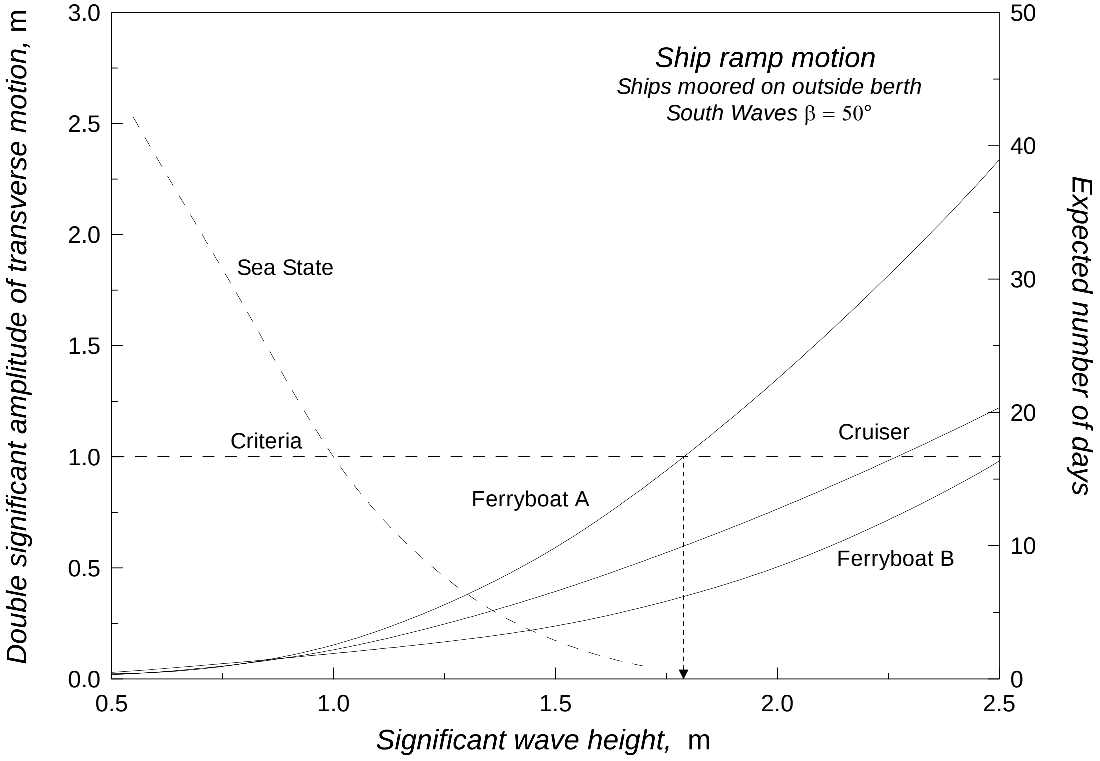

Fig. 10 Significant amplitude of ship ramp transversal motion

Slika 10. Značajna amplituda poprečnog gibanja rampe

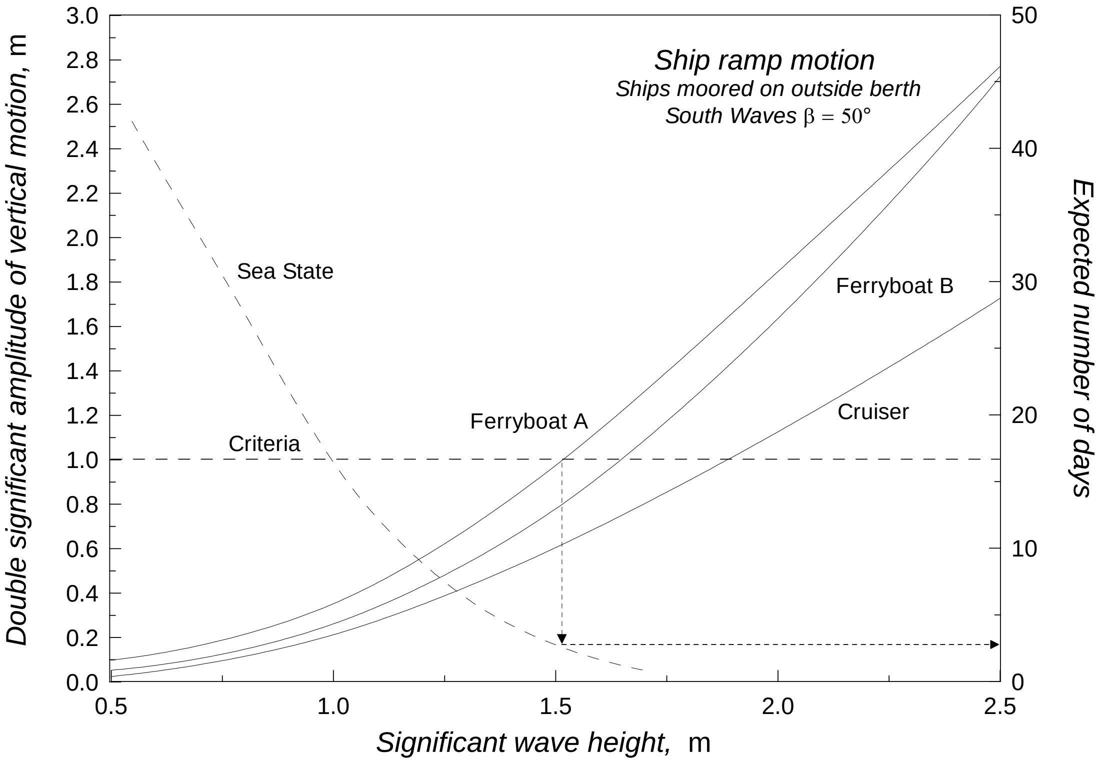

Fig. 11 Significant amplitude of ship ramp vertical motion

Slika 11. Značajna amplituda vertikalnog gibanja rampe

3. Conclusion

In this study, the operability of pier is discussed from by use of reliability approach and the probabilistic method to evaluate the significant amplitude of ramp absolute motion for moored ship. The horizontal and vertical absolute ramp motion significant amplitudes are chosen as criteria for safe working condition. The influence of mooring lines to ship motion is taken into account by appropriate restoring contributions from the mooring elements that are assembled into the body restoring matrix. The sea is described by Tabain spectrum.

As an example the operability of hypothetic outside berths in Adriatic Sea port is calculated taking into account the affect of sea waves as well as finite sea depth. As a result, based on chosen criteria limit, the number of non-operative days is calculated. The analysis showed that there is a good justification of outside berths constructions also because the main use of the berths is planed to be during summer months.

REFERENCES:

[1] Newman, J., N.: Marine Hydrodynamics; The MIT Press, Massachusetts, 1982.

[2] Det Norske Veritas: SESAM the Software Packages, DNVS Software, 2003.

[3] Senjanović, I., Čorić, V.: Strength Analysis of Ship Mooring System for Shallow Water Quay (I), Brodogradnja, Vol. 35, No. 4-5, p.p. 203-211, 1987.

[4] Senjanović, I., Čorić, V.: Strength Analysis of Ship Mooring System for Shallow Water Quay (II), Brodogradnja, Vol. 35, No. 6, p.p. 271-284, 1987.

[5] Tabain, T.: Standard Wind Wave Spectrum for the Adriatic Sea Revisited (1977-1997), Brodogradnja, Vol. 45, No. 4, p.p. 303-313, 1997.