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STATIC AND DYNAMIC ANALYSIS OF MARINE RISER

Summary

The analysis is concerned with static and dynamic behaviour of the explorational marine riser exposed to the sever environmental conditions. The most significant influence factors that can exert the loads on the riser structure are taken into account. The static analysis is based on a nonlinear mathematical model. The static equilibrium solution is used as an initial condition for the time domain dynamic analysis. Time domain analysis is based on a step-by-step numerical integration of the dynamic equilibrium equations. The wave load is given by the modified Morison equation. The frequency domain dynamic analysis is based on the linearised dynamic equilibrium equation at the static equilibrium position by application of stochastic linearisation of the hydrodynamic loading. Riser displacement, rotations, axial forces, bending moments, shear forces and natural modes and frequencies of free vibration are evaluated. Numerical procedure by the finite element method is utilized.

Key words: *marine risers, hydroelasticity, sea current, sea waves, finite element method*

STATIČKA I DINAMIČKA ANALIZA MORSKOG PODIZAČA

Sažetak

Istraživano je statičko i dinamičko ponašanje morskog podizača, izloženog nepovoljnim vremenskim uvjetima. U proračunu su razmatrani najznačniji uzročnici vanjskog opterećenja strukture podizača. Statička analiza temelji se na nelinearnom matematičkom modelu. Riješenje statičke ravnoteže uzeto je kao početni uvjet dinamičke analize u vremenskoj domeni. Analiza u vremenskoj domeni temelji se na postepenoj numeričkoj integraciji jednadžbi dinamičke ravnoteže. Valna opterećenja opisana su modificiranom Morisonovom jednadžbom. Dinamička analiza u frekventnoj domeni provedena je pomoću lineariziranih dinamičkih jednadžbi ravnoteže, te lineariziranih modela hidrodinamičkog opterećenja uz uvjet statičke ravnoteže na određenoj poziciji podizača. Određeni su pomaci i zaokreti podizača, uzdužne sile, momenti savijanja, tangencijalne sile, te modovi i frekvencije prirodnih vibracija strukture. U proračunu primjenjena je metoda konačnih elementa.

Ključne riječi: *podizači, hidroelastičnost , morske struje, valovi, metoda konačnih elemenata*

1. Introduction

Flexible marine risers are some of the most important components in floating production units for oil and gas. These slender flexible structures exhibit complex dynamic behaviour which has been investigated trough varies numerical and experimental methods in the last few decades [1]. The requirements placed on the drilling riser become increasingly severe as the offshore drilling operations develop into deeper waters. The response of deep water marine risers excited by sea current and sea surface waves is of great interest to the offshore industry. Precise prediction of the dynamic behaviour is of primary importance, therefore it is necessary to define all the significant sources of the environmental loads and the external forces [2].

This study is concerned with the drilling marine riser, fixed on its top and bottom end, and the analysis of static and dynamic behaviour of the riser exposed to a horizontal sea current, regular sea waves, static horizontal offset and the periodic horizontal motion of the platform (surge). The dynamic response is analysed as nonlinear and linearized problem in the time and frequency domain by using the finite element methods. All significant loads incorporated into the mathematical methods for the response analyses are shown in Figure 1. The analysis of marine riser static and dynamic behaviour is performed by computer programs developed for the slender, simply supported structures and adopted for the marine risers.

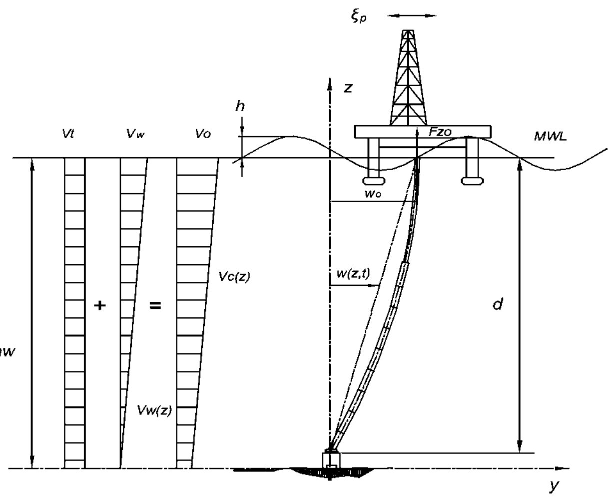


Fig. 1. Environmental loads exerted on the marine riser

Slika 1. Opterećenja morskog podizača

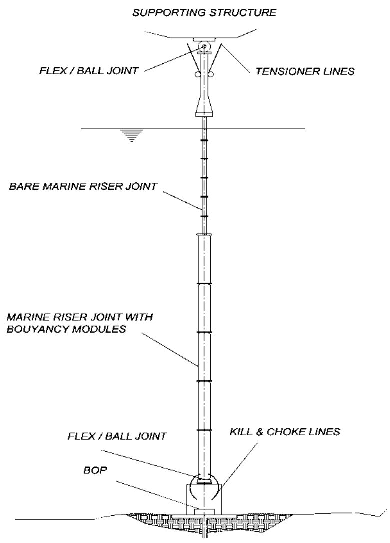


Fig. 2. Marine riser system

Slika 2. Sustav morskog podizača

2. Environmental condition and external forces

For estimating the external forces acting on a deep ocean pipe two general aspects had to be considered: the sub sea environmental characteristics and the external force produced by such environment. The environment is dominated by the vertical variation of sub sea current vector and seawater properties such a density, viscosity and temperature along with depth, and by strong winds and gusts that exert dominant aerodynamic forces on the riser and the supporting structure. The forces exerted on marine riser by surface waves are more significant then other sources of excitations. Internal waves, micro seismic waves, tides, tidal and volcanic waves are less significant and they are usually ignored in dynamic analysis [4].

2.1. Current drag force

The internal wave-generated current and tidal current are equally important for the analysis of the problem. The transverse load due to a sea current is determined according to the simplified current profile [3], which is assumed to coincide with the wind [5, 6, 7]. The tidal part is assumed to be constant along the riser, and the part of the current caused by the winds is approximated with the linear profile (Figure 1):

$$v_c(z) = v_t + v_w\left(\frac{z}{l}\right) \tag{1}$$

$$q(z) = \frac{1}{2}\rho_f C_D D\left(v_t + v_w\frac{z}{l}\right)^2 \tag{2}$$

C_D	– drag coefficient	v_t	– tidal current velocity
l	– riser length	$v_c(z)$	– sea current velocity
D	– diameter of the riser part	v_w	– wind current velocity

2.2. Static axial force

Static axial force consist of effective tension force, weight and buoyancy of the riser with sea water and drilling mud inside [4]:

$$N_s = F_{z0} - G_{ef}z \tag{3}$$

$$F_{z0} = F_{z1} - \frac{\pi}{4}g\rho_m\delta_m d_i^2 \tag{4}$$

F_{z0}	– effective tension force	F_{z1}	– riser tension force

2.3. Wave load

Regular waves are described in accordance with the theory of progressive sinusoidal wave. Horizontal wave load on the vertical cylinders is determined by the Morison equation that takes into account inertia and drag force, but neglects the diffraction component. In a case of small amplitude motion Morison equation takes the form [8]:

$$q(y,z,t) = C_I\frac{\partial^2\xi}{\partial t^2} + C_D\left|\frac{\partial\xi}{\partial t}\right|\frac{\partial\xi}{\partial t} \tag{5}$$

If a cylinder amplitude is of the same order of magnitude as the trajectory of a fluid particle, the relative motion of the fluid and cylinder has to be taken into account. The modified nonlinear Morison equation is dedicated to large motion analysis:

$$q(y,z,t) = C_I\left[\frac{\partial^2\xi}{\partial t^2} + \left(\frac{\partial\xi}{\partial t} - \frac{\partial w}{\partial t}\right)\frac{\partial^2\xi}{\partial y\partial t}\right] - m_a\frac{\partial^2 w}{\partial t^2} + C_D\left|\frac{\partial\xi}{\partial t} - \frac{\partial w}{\partial t}\right|\left(\frac{\partial\xi}{\partial t} - \frac{\partial w}{\partial t}\right) \tag{6}$$

where C_I, C_D and m_a are inertia, drag and added mass coefficients [9].

Linearised Morison equation is necessary to make the analysis of riser response in frequency domain possible. The convective load component of wave is neglected because it has minor influence on the riser response, drag component is linearised and the effect of large amplitude of riser motion is ignored. In such a way equation (6) takes the form:

$$q(z,y,t) = C_I\frac{\partial^2\xi}{\partial t^2} - m_a\frac{\partial^2 w}{\partial t^2} + \overline{C}_D\left(\frac{\partial\xi}{\partial t} - \frac{\partial w}{\partial t}\right) \tag{7}$$

Linearized drag coefficient for regular waves $\overline{C}_D$ is obtained from the equivalence of work of nonlinear and linearized drag force [10], where $\dot{r}_{\max}$ is amplitude of deterministic relative velocity.

$$\overline{C}_D = \frac{8}{3\pi} C_D \dot{r}_{\max} \tag{8}$$

3. Analysis mathematical models

3.1. Nonlinear response model

For the beam column model of riser exposed to transverse load due to regular waves and axial load due to weight, buoyancy and tension force, differential equation of motion is obtained [3, 8, 10]:

$$\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 w}{\partial z^2}\right) - \left(N_s + N_d\right)\frac{\partial^2 w}{\partial z^2} + G_{ef}\frac{\partial w}{\partial z} + C_s\frac{\partial w}{\partial t} + m_s\frac{\partial^2 w}{\partial t^2} = q(y,z,t) \tag{9}$$

Distributed mass m_s include structure mass and inside water and C_S is coefficient of structural damping. Dynamic axial force is result of buoyancy variation in a regular wave:

$$N_d = \rho g A\, \eta(d,y,t) \tag{10}$$

The modified Morison equation and the complete form of the differential equation of motion that is suitable for iteration time integration:

$$\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 w}{\partial z^2}\right) - N_s\frac{\partial^2 w}{\partial z^2} + G_{ef}\frac{\partial w}{\partial z} + C_s\frac{\partial w}{\partial t} + m_t\frac{\partial^2 w}{\partial t^2} = q(y,z,t)$$

$$q(y,z,t) = N_d\frac{\partial^2 w}{\partial x^2} + C_I\left[\frac{\partial^2 \xi}{\partial t^2} + \left(\frac{\partial \xi}{\partial t} - \frac{\partial w}{\partial t}\right)\frac{\partial^2 \xi}{\partial y \partial t}\right] + C_D\left|\frac{\partial \xi}{\partial t} - \frac{\partial w}{\partial t}\right|\left(\frac{\partial \xi}{\partial t} - \frac{\partial w}{\partial t}\right) \tag{11}$$

3.2. Linearized response model

Dynamic axial force and convective inertia components may be neglected and drag load can be linearized. The equation (11) may be solved in the frequency domain if the effect of the large amplitude riser motion is ignored. Linearized form of the differential equation of motion is:

$$\frac{\partial^2}{\partial z^2}\left(EI\frac{\partial^2 w}{\partial z^2}\right) - N_s\frac{\partial^2 w}{\partial z^2} + G_{ef}\frac{\partial w}{\partial z} + \left(C_s + \overline{C}_D\right)\frac{\partial w}{\partial t} + m_t\frac{\partial^2 w}{\partial t^2} = C_I\frac{\partial^2 \xi}{\partial t^2} + \overline{C}_D\frac{\partial \xi}{\partial t} \tag{12}$$

4. Analysis methods

4.1. Finite element method

The problems of static and dynamic behaviour of marine riser, formulated by equations of motion, may be solved by the finite element method. The marine riser is modeled by the string of beam finite elements having appropriate mass and stiffness characteristics.

The problem of the elastic deformation of the riser exposed to steady sea current and tension force is mathematically modeled by partial differential equation of static equilibrium [7,11]:

$$[K]\{d\} + [M]\{\ddot{d}\} = \{F\} \tag{13}$$

The stiffness matrix is general, consisting of bending stiffness and geometric stiffness matrix:

$$[K]=[K]_B+[K]_G \tag{14}$$

The modes of free undamped vibration may be calculated from the homogenous system of equations:

$$([K]_S+[K]_G)\{d\}+[M]\{\ddot{d}\}=\{0\} \tag{15}$$

Since the displacement vector is harmonic, the eigenvalue problem is formulated:

$$([K]_S+[K]_G-\omega^2[M])\{d\}=\{0\} \tag{16}$$

where $[m]$ is a global mass matrix and ω is a natural frequency of the system.

Partial nonlinear differential equation (11) and linearized differential equation (12) of riser motion in regular waves may be solved in space by the finite element method. The Pseudo-linear matrix equation represents equilibrium of nodal forces and compatibility of nodal displacements result in

$$[K]\{\delta\}+[C]\{\dot{\delta}\}+[M]\{\ddot{\delta}\}=\{F(t)\} \tag{17}$$

where

$[C]$, $[M]$ - damping and mass matrix

$\{\delta\},\{\dot{\delta}\},\ \{\ddot{\delta}\}$ - displacement, velocity and acceleration vectors

$\{F[t]\}$ - force vector dependent on load q

The matrix equation may be solved utilizing the mode superposition method:

$$\{\delta\}=[\phi]\{Z\} \tag{18}$$

where $[\phi]$ and $\{Z\}$ are undumped mode matrix and generalized displacement vector, respectively.

The equation (18) may be reduced to a set of coupled modal equations written in a pseudo-linear form:

$$\omega_i^2 Z_i+2\omega_i\xi_i\dot{Z}_i+\ddot{Z}_i=\varphi_i(t), \qquad i=1,2...N \tag{19}$$

$$\varphi_i(t)=\psi_i(t)-2\omega_i\sum_{j=1}^{N}\left(1-\delta_{ij}\right)\xi_{i,j}\dot{Z}_j \tag{20}$$

where N is number of d.o.f., $\delta_{i,j}$ is Kronecker symbol, ω_i is natural frequency of *i*–th mode, ξ_I coefficient of modal damping and $\psi_i\ (t)$ modal excitation.

4.2. Nonlinear solution in time domain

Nonlinear problem of marine riser response to a regular wave may be solved by time integration utilizing the harmonic acceleration method since it is unconditionally stable. The corresponding algorithm reads [8]:

$$\{Y\}_{i,j+1}^{(k+1)}=[T]\{Y\}_{i,j}+\{L\}\varphi_{i,j+1}^{(k)} \tag{21}$$

where $\{Y\}=\langle Z,\dot{Z},\ddot{Z}\rangle^T$ is response vector, $[T]$ integration transfer matrix and $\{L\}$ load operator. The subscript *i* (*1,2 ... N*) is the mode index, while *j* (*1,2,...N*) and *k* (0,1,2…) denote the time step and iteration step respectively. Since at nodes $w\in\{\delta\}$, it is necessary to determine $\{\delta\}$, $\{\dot{\delta}\}$ and $\{\ddot{\delta}\}$ in each time and iteration step using (18) and its derivates.

To start time integration by (21) the initial condition $\{\delta\}=\{\delta\}_0$ and $\{\dot{\delta}\}=\{\dot{\delta}\}_0$ at *t=0* have to be assumed.

4.3. Linearized solution in frequency domain

In the case of linearized differential equation (12) and small amplitude motion the marine riser response may be determined in the frequency domain. Governing modal load and the modal response are harmonic, therefore one may write:

$$\varphi_i(t) = \varphi_{si}\sin\omega_0 t + \varphi_{ci}\cos\omega_0 t \tag{22}$$

$$\varphi_i(t) = \varphi_{si}\sin\omega_0 t + \varphi_{ci}\cos\omega_0 t \tag{23}$$

By inserting (22) and (23) into (20) yields

$$Z_{si} = \frac{\left(1-\beta_i^2\right)\varphi_{si} + 2\xi_i\beta_i\varphi_{ci}}{\left[\left(1-\beta_i^2\right)^2 + \left(2\xi_i\beta_i\right)^2\right]\omega_i^2},\ Z_{ci} = \frac{\left(1-\beta_i^2\right)\varphi_{ci} - 2\xi_i\beta_i\varphi_{si}}{\left[\left(1-\beta_i^2\right)^2 + \left(2\xi_i\beta_i\right)^2\right]\omega_i^2} \tag{24}$$

where $\beta_i = \omega_0 / \omega_i, \quad i = 1,2,\ldots n.$

Displacement vector according to (18) is

$$\{\delta\} = \{\delta\}_s \sin\omega_0 t + \{\delta\}_c \cos\omega_0 t \tag{25}$$

The marine riser deflection is

$$w = W_s \sin\omega_0 t + W_c \cos\omega_0 t \tag{26}$$

5. Numerical example

Following the established algorithms, static and dynamic behaviour of marine riser is determined for the particular riser [7]. Numerical simulation are performed using discretized beam column model by computer programs developed for the slender, simply supported structures and adopted for the marine risers [8].

Table 2 Particulars of the marine riser

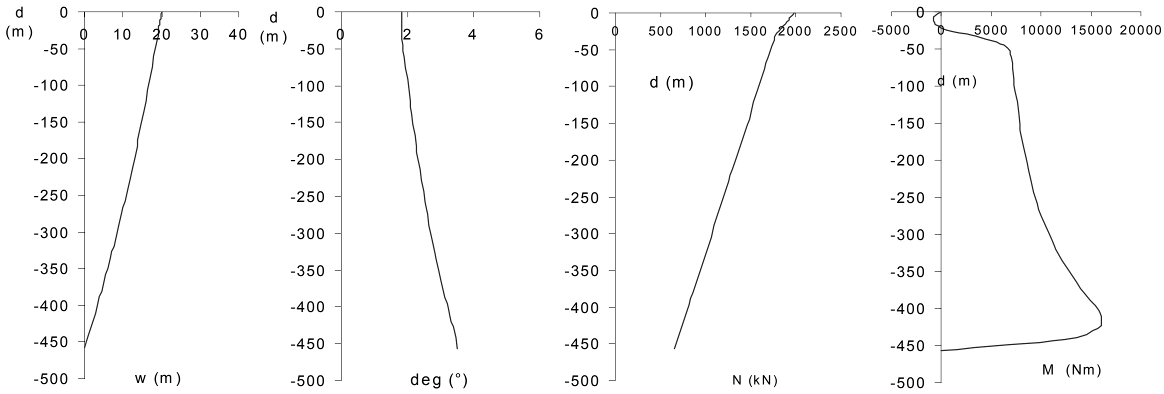
Tablica 2. Svojstva morskog podizača

Riser length	457 m	Modulus of elasticity, E	210 GPa
Outer diameter of riser	0.533 m	Density of water	1025 kg/m^3
Inner diameter of riser	0.508 m	Density of mud	2157 kg/m^3
Buoyancy module diameter	1.067 m	Water depth	470 m

5.1. Static analysis

Analysed static case includes the transverse static load exerted by the sea current and the platform offset of 20 m. Sea current velocity at the surface is 0.2 m/s and tidal current is 0.3 m/s. Maximum deflection angle at the riser bottom should not exceed 2-4 degrees during the operational period and 6-7 degrees during the sever whether condition.

According to this limitation safe factor for the tension force on top of the riser is determined. Drag coefficient for the riser joint is 1.1. Results of the numerical procedure show that the acceptable rotation angle (Fig. 3.b) is insured with the top tension force of 2000 kN (Fig. 3.c) for the safe factor 1.5. The largest static bending moment (15000 Nm) is close to the wellhead, at the sea depth of 420 m.



a) **b)** **c)** **d)**

Fig. 3. a) riser displacement, b) riser rotation c) axial force in riser d) static bending moment

Slika 3. a) pomak podizača, b) rotacija podizača, c) uzdužna sila, d) statički moment savijanja

5.2. Dynamic analysis

Analyzed dynamic case includes the load exerted by the time–varying wave forces and the platform surge motion. Surge motion is assumed to be sinusoidal with the same period as the wave, that is 14.6 s. Wave height is 9.5 m, surge amplitude is 4.2 m. Top tension is assumed to be constant throughout the wave cycle. Dynamic response of the riser structure model is shown in Figure 4. The largest deflection is at riser top and it corresponding with the surge motion amplitude (4.2 m). The highest bending moment amplitudes are at the sea depths of 390 m and 150 m (around 80 kNm). Amplitude of rotation angle at the marine riser bottom (lower ball joint) is 2.5 degrees. Rotation angle amplitudes, bending moments and shear forces determined in time domain show good agreement with the linearized solution.

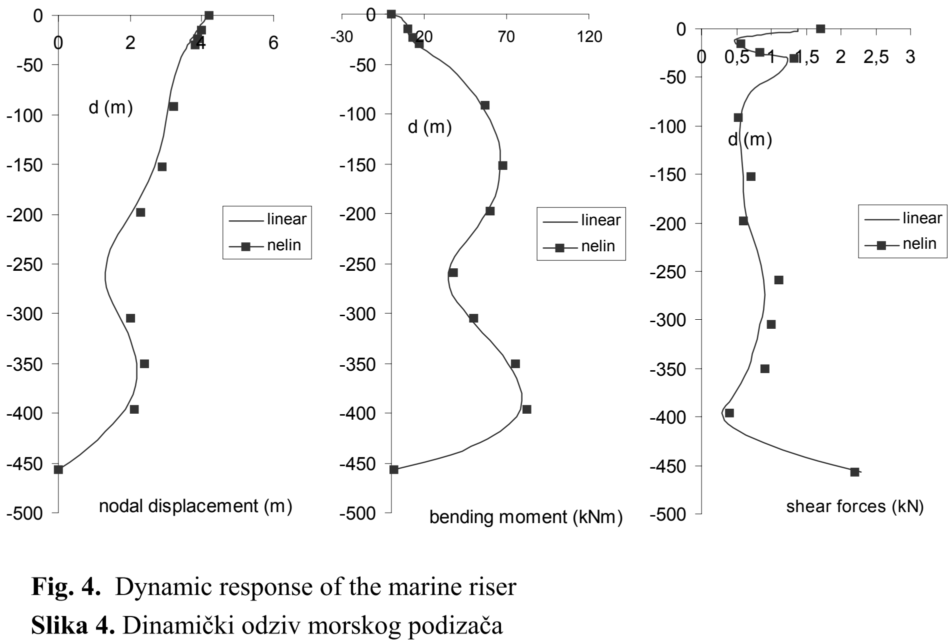


Fig. 4. Dynamic response of the marine riser

Slika 4. Dinamički odziv morskog podizača

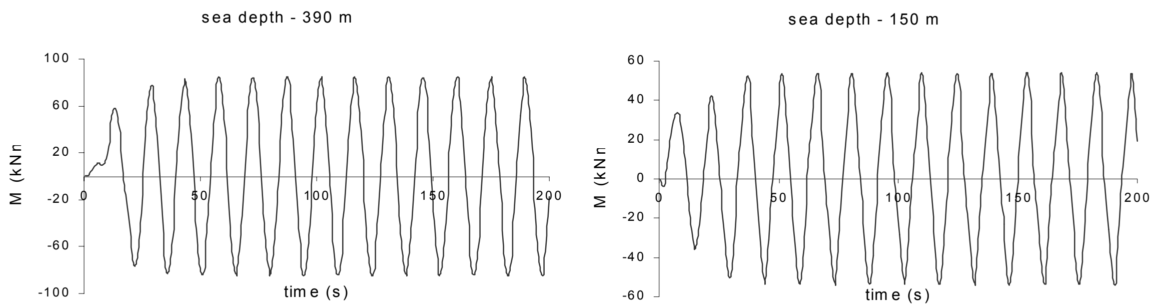


Fig. 5. Simulated bending moment of the marine riser at the critical positions

Slika 5. Simulacija momenta savijanja morskog podizača na kritičnim mjestima.

6. Conclusion

Marine riser is analysed with the computationally efficient method of evaluating the response of slender flexible structure subjected to static and dynamic excitations. The first level of marine riser analyses is static response due to a sea current and a platform offset. Riser displacement, rotations, axial forces, bending moments The tension force on top of the riser is determined with respect to the recommended rotation angle at the marine riser lower ball joint. Second level of riser analyses is dynamic behaviour due to the regular waves and platform surge motion. The problem of dynamic response is solved in linearized form in the frequency domain and in nonlinear form in the time domain. Regions on marine riser with highest bending moments and largest deflection amplitudes are determined. Both considered methods give similar results. Comparison of the results of static and dynamic analyses shows that the vessel motion and waves are the dominant source of riser bending stress.

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