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VERTICAL RESPONSE OF TLP WITH THE EFFECT OF ADDED MASS FLUCTUATION

Summary

The resulting motion in waves can be considered as a superposition of the motion of the body in still water and the forces on the restrained body. In this paper the effect of added mass fluctuation on the dynamic response of the leg (tether) of a tension leg platform (TLP) subjected to axial load at the top of the leg is presented. This effect is important when the amplitude of vibration is large as well as in fatigue life study of tethers. The leg of TLP is modeled as a vertical bar. The structural model is very simple but some complicated factors such as buoyancy and simulated ocean wave loads are considered. First order perturbation method is used in order to formulate and solve the problem. The differential equation is solved by means of non-harmonic Fourier expansion in terms of eigenfunctions obtained from a non-regular Sturm-Liouville system.

Keywords: added mass fluctuation, wave, TLP, vertical vibration, perturbation method, heave, continuous model, wave load

VERTIKALNI ODZIV PLATFORME SA ZATEZNIM KRACIMA UZIMAJUĆI U OBZIR FLUKTUACIJU DODATNE MASE

Sažetak

Rezultirajuće njihanje na valovima može se promatrati kao superpozicija prisilnih oscilacija tijela na mirnoj vodi i valnih opterećenja na nepokretno tijelo. U ovom je radu prikazan utjecaj fluktuiranja dodatne mase na dinamički odziv kraka platforme za zateznim kracima izložene uzdužnom opterećenju na vrhu kraka. Ovaj je utjecaj značajan pri velikim amplitudama vibriranja kao i pri analizama zamora materijala krakova. Krak platforme je modeliran kao vertikalni štap. Iako je strukturni model veoma jednostavan, razmatrani su neki komlicirani faktori, kao uzgon i simulirani oceanski valovi. Perturbacijska metoda prvog reda je korištena za formuliranje i rješenje problema. Diferencijalna jednadžba je riješena ne-harmonijskim Fourierovim razvojem u obliku vlastitih funkcija dobivenih iz nepravilnog Sturm-Liouville-ovog sustava.

Ključne riječi: fluktuacija dodatne mase, val, platforma sa zateznim kracima, vertikalne vibracije, perturbacijska metoda, poniranje, kontinuirani model, valno opterećenje

1. Introduction

Tension leg platforms (TLPs) are well-known structures for oil exploitation in deep water And are becoming increasingly popular for oil drilling at very deep water sites. These structures consist of semi-submersible platforms with sufficient buoyancy to develop the required tension in the tethers. The tension leg platform (TLP) is a moored floating structure whose buoyancy is more than its weight. The mooring system of TLP consists of number of tensioned tethers connected to the columns at the top and anchored to the seabed at the bottom. These tethers are vulnerable to failure due to fatigue produced by fluctuation of tension. Many studies have been carried out to understand the structural behavior of TLP and determine the effect of several parameters on dynamic response and average life time of the structure [1-6]. The tether system is a critical and basic component of the TLP. The most important point in the design of TLP is the pretension of the legs. The pretension causes that the platform behaves like a stiff structure with respect to the vertical degrees of freedom (heave, pitch and roll), whereas with respect to the horizontal degrees of freedom (surge, sway and yaw) it behaves as a floating structure. Therefore the periods of the vertical degrees of freedom are lower than the others. Among the various degrees of freedom, vertical motion (heave) is very important because of the direct effect on the stress fluctuation that leads to fatigue and fracture. Therefore the conceptual studies to understand the dynamic vertical response of TLP, can be useful for designers.

Simple models for heave response of tension leg platform under harmonic vertical load has been proposed [7]. The effect of added mass fluctuation on the heave response of tension leg platform has been investigated by using perturbation method for discrete [8] and continues model [9]. Added mass fluctuation has important effect on fatigue life of tethers [8]. Rossit et al. [10] presented an analytical solution for the dynamic response of the leg of TLP subjected to an axial suddenly applied load at one end. The applied load was constant and the effect of the buoyancy was not considered.

In this study the effect of added mass fluctuation in the case of vibration in still water for both free and forced vibration is discussed. A similar formulation can be developed for motion analysis of the restrained body in waves. The problem is solved by means of perturbation method [11-12].

2. Free vibration analysis

The resulting motion in waves can be seen as a superposition of the motion of the body in still water and the forces on the restrained body in waves (Fig. 1).

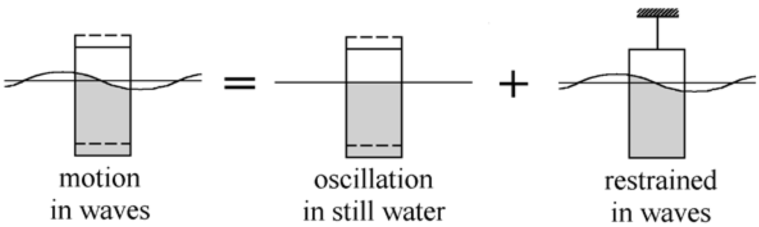

Fig 1. Superposition of Hydromechanical and Wave Loads [13]
Slika 1 Superpozicija hidromehaničkih i valnih opterećenja [13]

In this paper the oscillation in still water is considered. Also in the case of small diameter, the forces of restrained body in waves is similar to the oscillation in still water. The aim of this paper is the solution of the mentioned problem using a simple model considering buoyancy, added mass fluctuation and simulated ocean wave load. A vertical force is applied at the top of the leg as simulated ocean wave load. The problem is solved by means of non-harmonic Fourier expansion in terms of eigenfunctions obtained from a non-regular Sturm-Liouville system [10].

3. **Analytical solution of the model**

The structural model of the system has been shown in Fig. 2. The behavior of the system is described

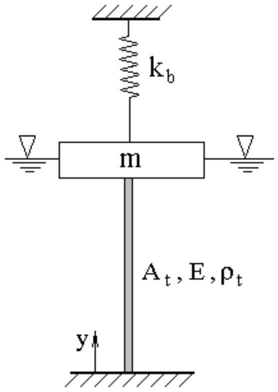

Fig 2. Dynamic structural model
Slika 2 Dinamički strukturni model

by the following differential equation

$$EA_t \frac{\partial^2 v}{\partial y^2} + F_h(t)\,\delta(y-l) = \left[\rho_t A_t + m(1+\varepsilon a v)\,\delta(y-l)\right]\frac{\partial^2 v}{\partial t^2} \tag{1}$$

where v is the axial deformation, E is the Young modulus of the tether material, A_t is the cross sectional area of the tether, $\rho_t = \rho - \rho_w$ is the effective density of tether, ρ is the density of tether and ρ_w is the density of water, l is length of it and δ denotes the Dirac delta function, ε is the added mass fluctuation parameter and a is the ratio of the added mass to the structural mass. The applied vertical load subjected to the mass m, is the generated wave load, $F_h(t) = \sum_{j=1}^{N} F_j \sin(\Omega_j t + \phi_j)$ obtained from the wave spectrum. The system is linear, therefore the solution of the Eq. (1) is carried out considering a single term input, $F_h(t) = F_0 \sin(\Omega t)$, and then the overall response of the system is evaluated by summation of all responses. The initial conditions of the differential equation are as follows

$$v(y,0) = 0 \tag{2a}$$

$$\frac{\partial v}{\partial t}(y,0) = 0 \tag{2b}$$

The mass distribution function is defined as

$$M(y,t) = \rho_t A_t + m\left[1 + \varepsilon a v(y,t)\right]\delta(y-l) \tag{3}$$

In the case of free vibration, Eq. (1) becomes

$$EA_t \frac{\partial^2 v}{\partial y^2} = M(y,t)\frac{\partial^2 v}{\partial t^2} \tag{4}$$

Considering the first order perturbation of the response [9], one has

$$v(y,t) = v_0(y,t) + \varepsilon v_1(y,t) \tag{5}$$

Substituting Eq. (5) in (1) results in

$$EA_t \frac{\partial^2 (v_0 + \varepsilon v_1)}{\partial y^2} + F_h(t)\delta(y-l) = \left[\rho_t A_t + m\left(1 + \varepsilon a(v_0 + \varepsilon v_1)\right)\delta(y-l)\right]\frac{\partial^2 (v_0 + \varepsilon v_1)}{\partial t^2} \tag{6}$$

According to the perturbation theory, one has two following equations

$$EA_t \frac{\partial^2 v_0}{\partial y^2} + F_h(t)\delta(y-l) = \left[\rho_t A_t + m\delta(y-l)\right]\frac{\partial^2 v_0}{\partial t^2} \tag{7}$$

$$EA_t \frac{\partial^2 v_1}{\partial y^2} + F_h'(t)\delta(y-l) = \left[\rho_t A_t + m\delta(y-l)\right]\frac{\partial^2 v_1}{\partial t^2} \tag{8}$$

where

$$F_h'(t) = -amv_0 \frac{\partial^2 v_0}{\partial t^2} \tag{9}$$

Equation. (7) can be solved assuming $M(y) = \rho_t A_t$, subjected to the boundary conditions

$$v_0(0,t) = 0 \tag{10a}$$

$$-EA_t \frac{\partial v_0}{\partial y}(l,t) - k_b v_0(l,t) = m\frac{\partial^2 v_0}{\partial t^2}(l,t) \tag{10b}$$

Using separation of variables, the eigenfunctions are determined as

$$Y_{n0} = \sin \alpha_{n0} y \tag{11}$$

and the frequency equation becomes

$$\alpha_{n0} l \tan \alpha_{n0} l = \frac{EA_t \alpha_{n0}^2 l}{m\alpha_{n0}^2 c^2 - k_b} \tag{12}$$

where $c^2 = E/\rho$, α_{n0} is the separation constant and $c\alpha_{n0}$ is the angular frequency. Defining $\rho_t A_t l = m_t$ (total mass of the tether) and $EA_t/l = k_t$ (axial stiffness of the tether), and substituting in Eq. (12), the frequency equation is reformed as

$$\tan \alpha_{n0} l = \frac{\alpha_{n0} l}{\frac{m}{m_t}\alpha_{n0}^2 l^2 - \frac{k_b}{k_t}} \tag{13}$$

The response of the tether subjected to axial load, can be expressed in terms of normal modes of the system as follows

$$v_0(y,t) = \sum_{n=1}^{\infty} Y_{n0}(y) T_{n0}(t) \tag{14}$$

Because of the orthogonality of the normal modes, it can be shown that

$$\int_0^l M(y) Y_{n0}(y) Y_{r0}(y) = 0 \ (n \neq r) \tag{15a}$$

$$\int_0^l M(y) Y_{n0}(y) Y_{r0}(y) = H_{r0} \ (n = r) \tag{15b}$$

where

$$H_{r0} = \frac{m_t}{2} + Y_{r0}^2(l)\left(\frac{k_b}{2\alpha_{r0}^2 c^2} + \frac{m}{2}\right) \tag{16}$$

Multiplying Eq. (7) by $Y_r(y)dy$ and integrating between 0 and l, one obtains

$$EA_t \int_0^l Y_{r0} \left(\sum Y''_{n0} T_{n0} \right) dy + F_h(t) \int_0^l Y_{r0} \delta(y-l) dy = \int_0^l M(y) Y_{r0} \left(\sum Y_{n0} \ddot{T}_{n0} \right) dy \quad (17)$$

or

$$EA_t \int_0^l Y_{r0} \left(\sum Y''_{n0} T_{n0} \right) dy + F_h(t) Y_{r0}(l) = H_{r0} \ddot{T}_{r0} \quad (18)$$

Since Y_{n0} satisfies (11), one has

$$Y''_{n0} = -\frac{M(y)}{EA_t} c^2 \alpha_{n0}^2 Y_{n0} \quad (19)$$

Substituting Eq. (19) in (18) and applying (15) results in

$$\ddot{T}_{n0} + c^2 \alpha_{n0}^2 T_{n0} = \frac{Y_{n0}(l)}{H_{n0}} F_h(t) \quad (20)$$

or

$$\ddot{T}_{n0} + c^2 \alpha_{n0}^2 T_{n0} = \frac{Y_{n0}(l)}{H_{n0}} F_0 \sin(\Omega t)$$

Solving the above differential equation, gives

$$T_{n0}(t) = A_{n0} \cos c\alpha_{n0} t + B_{n0} \sin c\alpha_{n0} t + \frac{F_0}{c^2 \alpha_{n0}^2 - \Omega^2} \frac{Y_{n0}(l)}{H_{n0}} \sin(\Omega t) \quad (21)$$

Initial conditions result in

$$A_{n0} = 0 \; ; \; B_{n0} = \frac{F_0}{c^2 \alpha_{n0}^2 - \Omega^2} \frac{\Omega}{c\alpha_{n0}} \frac{Y_{n0}(l)}{H_{n0}}$$

and

$$T_{n0}(t) = \frac{F_0}{c^2 \alpha_{n0}^2 - \Omega^2} \frac{Y_{n0}(l)}{H_{n0}} \left(-\frac{\Omega}{c\alpha_{n0}} \sin c\alpha_{n0} t + \sin(\Omega t) \right) \quad (22)$$

Now the dynamic response of the tether is given by

$$v_0(y,t) = \sum_{n=1}^{\infty} \frac{F_0}{c^2 \alpha_{n0}^2 - \Omega^2} \frac{Y_{n0}(l) \sin \alpha_{n0} y \left(-\frac{\Omega}{c\alpha_{n0}} \sin c\alpha_{n0} t + \sin(\Omega t) \right)}{\frac{m_t}{2} + Y_{r0}^2(l) \left(\frac{k_b}{2\alpha_{r0}^2 c^2} + \frac{m}{2} \right)} \quad (23)$$

Equation. (8) can be solved assuming $M(y) = \rho_t A_t$ subjected to the boundary conditions as follows

$$v_1(0,t) = 0 \quad (24a)$$

$$-EA_t \frac{\partial v_1}{\partial y}(l,t) - k_b v_1(l,t) = m \frac{\partial^2 v_1}{\partial t^2}(l,t) \quad (24b)$$

Using separation of variables, the eigenfunctions are determined as

$$Y_{n1} = \sin \alpha_{n1} y \quad (25)$$

and the frequency equation becomes

$$\tan \alpha_{n1} l = \frac{\alpha_{n1} l}{\frac{m}{m_t} \alpha_{n1}^2 l^2 - \frac{k_b}{k_t}} \quad (26)$$

where α_{n1} is the separation constant.

The response of the tether subjected to the axial load, can be expressed in terms of normal modes of the system as follows

$$v_1(y,t)=\sum_{n=1}^{\infty}Y_{n1}(y)T_{n1}(t) \tag{27}$$

where

$$H_{r1}=\frac{m_t}{2}+Y_{r1}^2(l)\left(\frac{k_b}{2\alpha_{r1}^2c^2}+\frac{m}{2}\right) \tag{28}$$

Like previous, one has

$$Y_{n1}''=-\frac{M(y)}{EA_t}c^2\alpha_{n1}^2Y_{n1} \tag{29}$$

$$\ddot{T}_{n1}+c^2\alpha_{n1}^2T_{n1}=\frac{Y_{n1}(l)}{H_{n1}}F_h'(t) \tag{30a}$$

Substituting Eq. (9) in (30a) results in

$$\ddot{T}_{n1}+c^2\alpha_{n1}^2T_{n1}=am\frac{Y_{n1}(l)}{H_{n1}}Y_{n0}^2(l)T_{n0}\ddot{T}_{n0} \tag{30b}$$

Regarding Eq. (22), the right hand side of Eq. (30b) becomes

$$T_{n0}\ddot{T}_{n0}=\Omega c\alpha_{n0}\left(\frac{F_0}{c^2\alpha_{n0}^2-\Omega^2}\frac{Y_{n0}(l)}{H_{n0}}\right)^2\left[\frac{\Omega}{c\alpha_{n0}}\left(-\sin^2 c\alpha_{n0}t-\sin^2\Omega t\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\sin c\alpha_{n0}t\sin\Omega t\right] \tag{31}$$

After some mathematical calculation, Eq. (31) is reformed as

$$T_{n0}\ddot{T}_{n0}=\frac{\Omega c\alpha_{n0}}{2}\left(\frac{F_0}{c^2\alpha_{n0}^2-\Omega^2}\frac{Y_{n0}(l)}{H_{n0}}\right)^2\left[\frac{\Omega}{c\alpha_{n0}}\left(\cos 2c\alpha_{n0}t+\cos 2\Omega t-2\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\cos(c\alpha_{n0}-\Omega)t-\cos(c\alpha_{n0}+\Omega)t\right)\right] \tag{32}$$

Now Eq. (30b) can be rewritten as

$$\ddot{T}_{n1}+c^2\alpha_{n1}^2T_{n1}=am\Omega c\alpha_{n0}\frac{Y_{n1}(l)Y_{n0}^2(l)}{2H_{n1}}\left(\frac{F_0}{c^2\alpha_{n0}^2-\Omega^2}\frac{Y_{n0}(l)}{H_{n0}}\right)^2\times\left[\frac{\Omega}{c\alpha_{n0}}\left(\cos 2c\alpha_{n0}t+\cos 2\Omega t-2\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\cos(c\alpha_{n0}-\Omega)t-\cos(c\alpha_{n0}+\Omega)t\right)\right] \tag{33}$$

Solving the differential Eq. (30), one obtains

$$T_{n1}(t)=A_{n1}\cos c\alpha_{n1}t+B_{n1}\sin c\alpha_{n1}t+am\Omega c\alpha_{n0}\frac{Y_{n1}(l)Y_{n0}^2(l)}{2H_{n1}}\left(\frac{F_0}{c^2\alpha_{n0}^2-\Omega^2}\frac{Y_{n0}(l)}{H_{n0}}\right)^2\times\left[\frac{\Omega}{c\alpha_{n0}}\left(-\frac{2}{c^2\alpha_{n0}^2}-\frac{\cos 2c\alpha_{n0}t}{3c^2\alpha_{n0}^2}+\frac{\cos 2\Omega t}{c^2\alpha_{n0}^2-4\Omega^2}\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\frac{\cos(c\alpha_{n0}-\Omega)t}{\Omega(2c\alpha_{n0}-\Omega)}+\frac{\cos(c\alpha_{n0}+\Omega)t}{\Omega(2c\alpha_{n0}+\Omega)}\right)\right] \tag{34}$$

From initial conditions, one has

$B_{n1}=0$ and

$$A_{n1} = -am\Omega c\alpha_{n0}\frac{Y_{n1}(l)Y_{n0}^2(l)}{2H_{n1}}\left(\frac{F_0}{c^2\alpha_{n0}^2-\Omega^2}\frac{Y_{n0}(l)}{H_{n0}}\right)^2\left[\frac{\Omega}{c\alpha_{n0}}\left(-\frac{7}{3c^2\alpha_{n0}^2}+\frac{1}{c^2\alpha_{n0}^2-4\Omega^2}\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\frac{4c\alpha_{n0}}{\Omega(4c^2\alpha_{n0}^2-\Omega^2)}\right)\right]$$

and therefore

$$T_{n1}(t) = -am\Omega c\alpha_{n0}\frac{Y_{n1}(l)Y_{n0}^2(l)}{2H_{n1}}\left(\frac{F_0}{c^2\alpha_{n0}^2-\Omega^2}\frac{Y_{n0}(l)}{H_{n0}}\right)^2\times$$

$$\left\{\left[\frac{\Omega}{c\alpha_{n0}}\left(-\frac{7}{3c^2\alpha_{n0}^2}+\frac{1}{c^2\alpha_{n0}^2-4\Omega^2}\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\frac{4c\alpha_{n0}}{\Omega(4c^2\alpha_{n0}^2-\Omega^2)}\right)\right]\cos c\alpha_{n1}t-\right. \tag{35}$$

$$\left.\left[\frac{\Omega}{c\alpha_{n0}}\left(-\frac{2}{c^2\alpha_{n0}^2}-\frac{\cos 2c\alpha_{n0}t}{3c^2\alpha_{n0}^2}+\frac{\cos 2\Omega t}{c^2\alpha_{n0}^2-4\Omega^2}\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\frac{\cos(c\alpha_{n0}-\Omega)t}{\Omega(2c\alpha_{n0}-\Omega)}+\frac{\cos(c\alpha_{n0}+\Omega)t}{\Omega(2c\alpha_{n0}+\Omega)}\right)\right]\right\}$$

Substituting Eqs (25) and (35) in (27), v_1 is determined as

$$v_1(y,t) = \sum_{n=1}^{\infty} -am\Omega c\alpha_{n0}\frac{Y_{n1}(l)Y_{n0}^2(l)}{2H_{n1}}\left(\frac{F_0}{c^2\alpha_{n0}^2-\Omega^2}\frac{Y_{n0}(l)}{H_{n0}}\right)^2\sin\alpha_{n1}y\times$$

$$\left\{\left[\frac{\Omega}{c\alpha_{n0}}\left(-\frac{7}{3c^2\alpha_{n0}^2}+\frac{1}{c^2\alpha_{n0}^2-4\Omega^2}\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\frac{4c\alpha_{n0}}{\Omega(4c^2\alpha_{n0}^2-\Omega^2)}\right)\right]\cos c\alpha_{n1}t-\right. \tag{36}$$

$$\left.\left[\frac{\Omega}{c\alpha_{n0}}\left(-\frac{2}{c^2\alpha_{n0}^2}-\frac{\cos 2c\alpha_{n0}t}{3c^2\alpha_{n0}^2}+\frac{\cos 2\Omega t}{c^2\alpha_{n0}^2-4\Omega^2}\right)+\left(1+\frac{\Omega^2}{c^2\alpha_{n0}^2}\right)\left(\frac{\cos(c\alpha_{n0}-\Omega)t}{\Omega(2c\alpha_{n0}-\Omega)}+\frac{\cos(c\alpha_{n0}+\Omega)t}{\Omega(2c\alpha_{n0}+\Omega)}\right)\right]\right\}$$

From Eqs (13) and (26) one has

$\alpha_{n0} = \alpha_{n1} = \alpha_n$

Now the dynamic response of the tether becomes

$$v(y,t) = \sum_{n=1}^{\infty}\sum_{j=1}^{N}\frac{F_j}{c^2\alpha_n^2-\Omega_j^2}\frac{\sin\alpha_n l\sin\alpha_n y\left(-\frac{\Omega_j}{c\alpha_n}\sin c\alpha_n t+\sin\Omega_j t\right)}{\frac{m_t}{2}+\sin^2\alpha_n l\left(\frac{k_b}{2\alpha_n^2c^2}+\frac{m}{2}\right)}-$$

$$\varepsilon\sum_{n=1}^{\infty}\sum_{j=1}^{N}\frac{1}{2}am\Omega_j c\alpha_n\left(\frac{F_j}{c^2\alpha_n^2-\Omega_j^2}\right)^2\frac{\sin^5\alpha_n l\sin\alpha_n y}{\left[\frac{m_t}{2}+\sin^2\alpha_n l\left(\frac{k_b}{2\alpha_n^2c^2}+\frac{m}{2}\right)\right]^3}\times \tag{37}$$

$$\left\{\left[\frac{\Omega_j}{c\alpha_n}\left(-\frac{7}{3c^2\alpha_n^2}+\frac{1}{c^2\alpha_n^2-4\Omega_j^2}\right)+\left(1+\frac{\Omega_j^2}{c^2\alpha_n^2}\right)\left(\frac{4c\alpha_n}{\Omega_j(4c^2\alpha_n^2-\Omega_j^2)}\right)\right]\cos c\alpha_n t-\right.$$

$$\left.\left[\frac{\Omega_j}{c\alpha_n}\left(-\frac{2}{c^2\alpha_n^2}-\frac{\cos 2c\alpha_n t}{3c^2\alpha_n^2}+\frac{\cos 2\Omega_j t}{c^2\alpha_n^2-4\Omega_j^2}\right)+\left(1+\frac{\Omega_j^2}{c^2\alpha_n^2}\right)\left(\frac{\cos(c\alpha_n-\Omega_j)t}{\Omega_j(2c\alpha_n-\Omega_j)}+\frac{\cos(c\alpha_n+\Omega_j)t}{\Omega_j(2c\alpha_n+\Omega_j)}\right)\right]\right\}$$

and the dynamic stress of the tether is calculated as follows

$$\sigma(y,t) = EA_t\frac{\partial v}{\partial y}(y,t) \tag{38}$$

Similarly second order perturbation can be used to determine the structural response.

4. An example

A numerical study has been carried out to understand the effect of parameters ε and a on the amplitude and frequency of vibration. Here only the first eigenfunstion is considered for simplicity. The structural properties are assumed. Also a harmonic wave excitation with frequency $\omega = 2\pi$ and amplitude equal to 1 is considered. Fig. (3) shows the response time history and phase plane for $\varepsilon = 0.5$ and a= 0.25, 0.5 and 0.75. It is observed that the linear solution is not close to the perturbed solutions. Also the solution of first and second order perturbations are approximately close together. For small values of a, the difference between the first and second order perturbations is small and it increases with increasing a. Considering second order perturbation leads to change in the frequency of vibration. Time history of response for $\varepsilon = a = 0.75$, is shown in Fig. (4). It is seen that there is a slowly varying shift in the period of vibration in the case of second order perturbation.

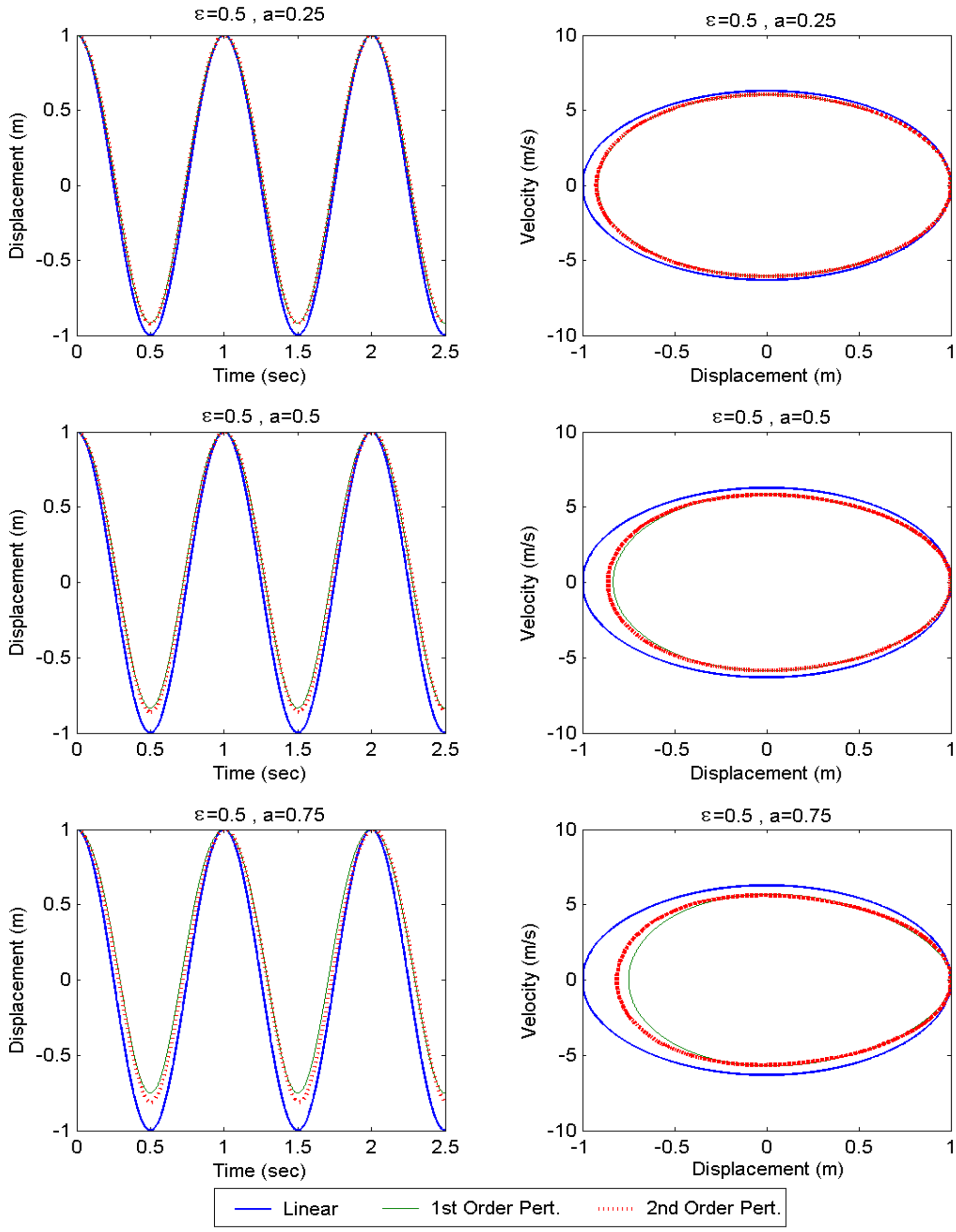

Fig. 3 Response and Phase Plane of System ($\varepsilon = 0.5$ and a= 0.25, 0.5, 0.75)

Slika 3 Vremenska povijest i fazna ravnina odziva ($\varepsilon = 0.5$ and a= 0.25, 0.5, 0.75)

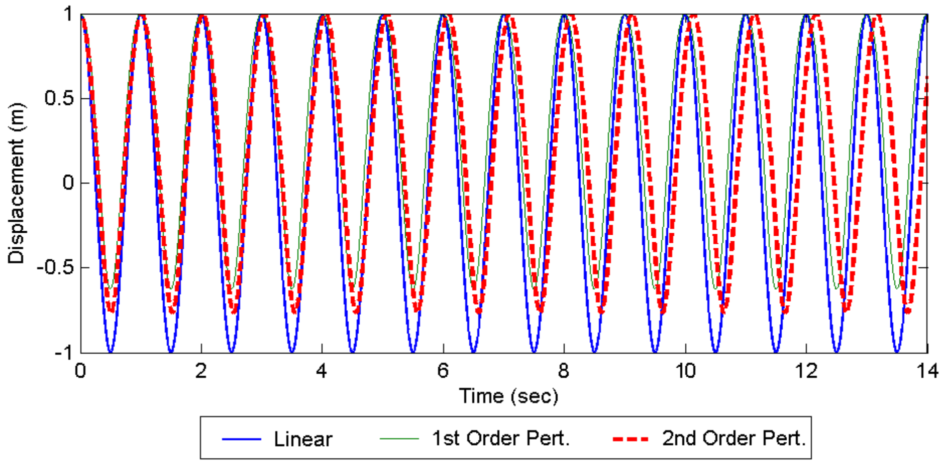

Fig. 4 Time History Response of System ($\varepsilon = a = 0.75$)

Slika 4 Vremenska povijest odziva ($\varepsilon = a = 0.75$)

5. Conclusion

The analytical solution of the tether response of TLP were presented for a continuous model considering the buoyancy and the effect of added mass fluctuation under the load simulated as ocean wave. First order perturbation method was used to solve the differential equation approximately. The presented solution gives a conceptual view of the heave response of TLP under sea wave loads. It can also be used in analytical study on fatigue life of tethers.

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