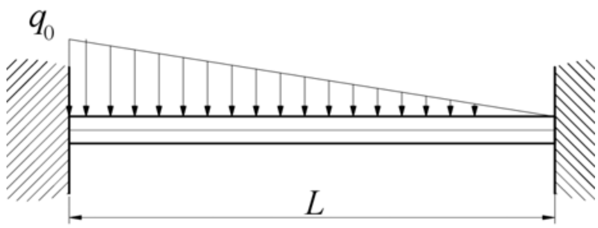
Rješenja 1. kolokvija od 19.04.2011. iz kolegija
NUMERIČKE METODE U STROJARSTVU

1. Izvesti izraz za rješavanje problema štapa opterećnog na savijanje Galerkinovom metodom. Za nosač zadan i opterećen prema slici potrebno je primjenom Galerkinove metode postaviti jednadžbe za određivanje pomaka. Za pretpostavljenu funkciju pomaka uzeti:

$$\bar{w} = a_1 \left(\frac{x}{L}\right)^2 \left(1 - \frac{x}{L}\right)^2 .$$

Zadano: $L, EI = konst., q_0$.



○Geometrijski rubni uvjeti:

$$\bar{w}(x=0)=0,\ \bar{w}(x=L)=0,\ \frac{\mathrm{d}\bar{w}}{\mathrm{d}x}(x=0)=0,\ \frac{\mathrm{d}\bar{w}}{\mathrm{d}x}(x=L)=0 .$$

$\bar{w} = a_1 \left(\frac{x}{L}\right)^2 \left(1 - \frac{x}{L}\right)^2 :\ 0 = a_1 (0)^2 (1)^2$ zadovoljava, $0 = a_1 (1)^2 (0)^2$ zadovoljava.

Derivacija pretpostavljene funkcije je

$$\frac{\mathrm{d}\bar{w}}{\mathrm{d}x} = a_1 \left[2\left(\frac{x}{L}\right)\frac{1}{L}\left(1-\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^2\left(1-\frac{x}{L}\right)\left(-\frac{1}{L}\right)\right] : 0 = a_1 \left[2(0)\frac{1}{L}(1)^2 + 2(0)^2(1)\left(-\frac{1}{L}\right)\right]$$

zadovoljava, $0 = a_1 \left[2(1)\frac{1}{L}(0)^2 + 2(1)^2(0)\left(-\frac{1}{L}\right)\right]$ zadovoljava. Predložena funkcija zadovoljava rubne uvjete pomaka.

Izvod jednadžbe Galerkinove metode za riješavanje problema savijanja tankih štapova je u knjizi Uvod u numeričke metode u strojarstvu, Jurica Sorić, stranica 61. Jednadžba glasi:

$$\int_0^l \left(EI_y \frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2}\frac{\mathrm{d}^2 f_i}{\mathrm{d}x^2} - q_z f_i \right)\mathrm{d}x + \left[f_i EI_y \frac{\mathrm{d}^3\bar{w}}{\mathrm{d}x^3}\right]_0^l - \left[\frac{\mathrm{d}f_i}{\mathrm{d}x} EI_y \frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2}\right]_0^l = 0 .$$

Težinska funkcija

$$f_1 = \left(\frac{x}{L}\right)^2\left(1-\frac{x}{L}\right)^2 .$$

$$\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2} = a_1\left[4\left(\frac{x}{L}\right)\frac{1}{L}\left(1-\frac{x}{L}\right)\left(-\frac{1}{L}\right) + 4\left(\frac{x}{L}\right)\frac{1}{L}\left(1-\frac{x}{L}\right)\left(-\frac{1}{L}\right) + 2\left(\frac{1}{L}\right)^2\left(1-\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^2\left(-\frac{1}{L}\right)^2\right] = .$$

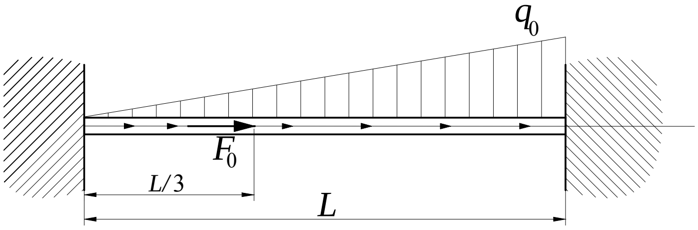
$$a_1\left(2\left(\frac{x}{L}\right)^2\left(\frac{1}{L}\right)^2 - 8\left(\frac{x}{L}\right)\left(1-\frac{x}{L}\right)\left(\frac{1}{L}\right)^2 + 2\left(\frac{1}{L}\right)^2\left(1-\frac{x}{L}\right)^2\right)$$

$$\frac{\mathrm{d}f_1}{\mathrm{d}x} = 2\left(\frac{x}{L}\right)\frac{1}{L}\left(1-\frac{x}{L}\right)^2 + 2\left(\frac{x}{L}\right)^2\left(1-\frac{x}{L}\right)\left(-\frac{1}{L}\right),$$

$$\frac{\mathrm{d}^2 f_1}{\mathrm{d}x^2} = 2\left(\frac{x}{L}\right)^2\left(\frac{1}{L}\right)^2 - 8\left(\frac{x}{L}\right)\left(1-\frac{x}{L}\right)\left(\frac{1}{L}\right)^2 + 2\left(\frac{1}{L}\right)^2\left(1-\frac{x}{L}\right)^2 .$$

$$EI_y\int_0^L a_1\left(2\left(\frac{x}{L}\right)^2\left(\frac{1}{L}\right)^2-8\left(\frac{x}{L}\right)\left(1-\frac{x}{L}\right)\left(\frac{1}{L}\right)^2+2\left(\frac{1}{L}\right)^2\left(1-\frac{x}{L}\right)^2\right)$$
$$\left(2\left(\frac{x}{L}\right)^2\left(\frac{1}{L}\right)^2-8\left(\frac{x}{L}\right)\left(1-\frac{x}{L}\right)\left(\frac{1}{L}\right)^2+2\left(\frac{1}{L}\right)^2\left(1-\frac{x}{L}\right)^2\right)\mathrm{d}x \quad .$$
$$-q_0\int_0^L\left(\frac{x}{L}\right)^2\left(1-\frac{x}{L}\right)^2\,\mathrm{d}x=0$$

2. Za štap zadan i opterećen prema slici potrebno je primjenom Rayleigh-Ritzove metode odrediti raspodjelu pomaka i uzdužne sile. Za pretpostavljenu funkciju pomaka uzeti: $\bar{u}=a_1+a_2x+a_3x^2$.
Zadano: $L, AE=\text{konst.}, q_0, F_0=q_0L$.



○Geometrijski rubni uvjet:

$$\bar{u}(x=0)=0, \bar{u}(x=L)=0 \; .$$

Pretpostavljena funkcija pomaka zadovoljava geometrijske rubne uvjete:
$0=a_1+a_2 0+a_3 0^2 \Rightarrow a_1=0; 0=a_2L+a_3L^2 \Rightarrow a_2=-a_3L$.
Konačno je pretpostavljena funkcija pomaka oblika

$$\bar{u}=a_3x^2-a_3Lx=a_3\left(x^2-Lx\right).$$

Funkcional za zadani štap je

$$\Pi=\frac{1}{2}AE\int_0^L\left(\frac{\mathrm{d}\bar{u}}{\mathrm{d}x}\right)^2dx-\int_0^L q_x\bar{u}dx-\left[F_0\bar{u}\right]_{\frac{L}{3}} \; .$$

$$\Pi=\frac{1}{2}AE\int_0^L\left(a_3\left(2x-L\right)\right)^2dx-\int_0^L q_0\frac{x}{L}a_3\left(x^2-Lx\right)dx-\left[F_0a_3\left(x^2-Lx\right)\right]_{\frac{L}{3}} .$$

$$\Pi=\frac{1}{2}AEa^2{}_3\left(L^3-2L^3+\frac{4L^3}{3}\right)-q_0a_3\left(\frac{L^3}{4}-\frac{L^3}{3}\right)-F_0a_3\left(\frac{L^2}{9}-\frac{L^2}{3}\right).$$

$$\Pi=\frac{1}{2}AEa^2{}_3\left(\frac{L^3}{3}\right)+q_0a_3\left(\frac{L^3}{12}\right)+q_0a_3\left(\frac{2L^3}{9}\right).$$

$$\frac{\partial\Pi}{\partial a_3}=0 \; .$$

$$\frac{\partial\Pi}{\partial a_3}=AEa_3\left(\frac{L^3}{3}\right)+q_0\left(\frac{11L^3}{36}\right)=0\,,$$

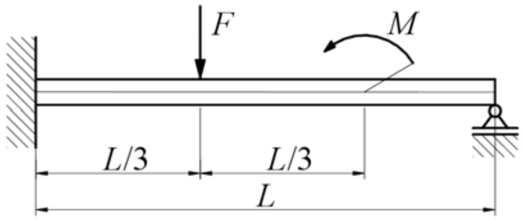
$$a_3=-\frac{q_0}{AE}\frac{11}{12}\,,\;\; \bar{u}=-\frac{q_0}{AE}\frac{11}{12}\left(x^2-Lx\right).$$

Uzdužna sila:

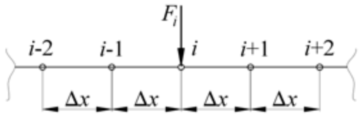
$$\bar{N}_x=AE\frac{\mathrm{d}\bar{u}}{\mathrm{d}x}\;,\;\bar{N}_x=-q_0\frac{11}{12}\left(2x-L\right).$$

3. Za nosač zadan i opterećen prema slici potrebno je primjenom metode konačnih razlika izvesti sustav jednadžbi u matričnom obliku za određivanje nepozantih progiba. Provesti diskretizaciju s $n = 7$ jednako razmaknutih čvorova. Izvesti izraze za potrebne rubne uvjete. Izvesti izraz za izračunavanje reakcije u zglobnom osloncu.

Zadano: $L, EI = \text{konst.}, F, M = FL$.



Zadatku mogu korisno poslužiti sljedeće relacije:



$$w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+1} + w_{i+2} = \frac{(\Delta x)^3}{EI} F_i,$$

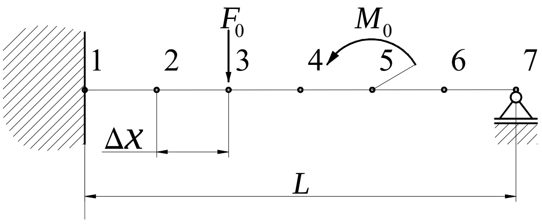
○Geometrijski rubni uvjeti:

$$\overline{w}(x=0)=0,\ \overline{w}(x=L)=0,\ \frac{\mathrm{d}\overline{w}}{\mathrm{d}x}(x=0)=0.$$

○Prirodni rubni uvjeti:

$$M(x=L)=0.$$

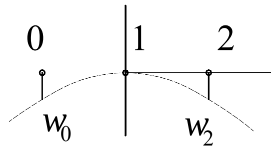
Proračunski model:



$$\Delta x = \frac{L}{6}$$

Diferencijske jednadžbe po čvorovima:

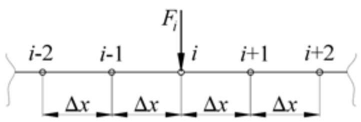
Čvor 2:



$$w_{i-2} = w_i, \quad 7w_i - 4w_{i+1} + w_{i+2} = \frac{(\Delta x)^3}{EI} F_i \ .$$

$$7w_2 - 4w_3 + w_4 = 0 .$$

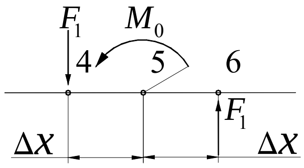
Čvor 3:



$$w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+1} + w_{i+2} = \frac{(\Delta x)^3}{EI} F_i,$$

$$-4w_2 + 6w_3 - 4w_4 + w_5 = \frac{(\Delta x)^3}{EI}(F_0).$$

Čvor 4:



$$M_0 = F_1 2\Delta x \rightarrow F_1 = \frac{M_0}{2\Delta x} = \frac{6F_0\Delta x}{2\Delta x} = 3F_0 .$$

$$w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+1} + w_{i+2} = \frac{(\Delta x)^3}{EI} F_i$$

$$w_2 - 4w_3 + 6w_4 - 4w_5 + w_6 = \frac{(\Delta x)^3}{EI} 3F_0 .$$

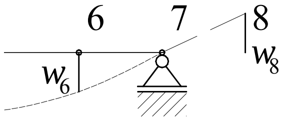
$$w_{i-2} = -w_i , \quad 5w_i - 4w_{i+1} + w_{i+2} = \frac{(\Delta x)^3}{EI} F_i \ .$$

Čvor 5:

$$w_{i-2} - 4w_{i-1} + 6w_i - 4w_{i+1} + w_{i+2} = \frac{(\Delta x)^3}{EI} F_i$$

$$w_3 - 4w_4 + 6w_5 - 4w_6 = 0 .$$

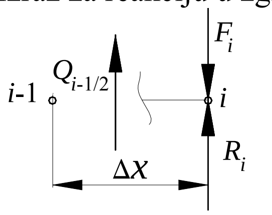
Čvor 6:



$$w_{i+2} = -w_i , \ 5w_6 - 4w_5 + w_4 = \frac{(\Delta x)^3}{EI}(-3F_0) .$$

$$\begin{bmatrix} 7 & -4 & 1 & 0 & 0 \\ -4 & 6 & -4 & 1 & 0 \\ 1 & -4 & 6 & -4 & 1 \\ 0 & 1 & -4 & 6 & -4 \\ 0 & 0 & 1 & -4 & 5 \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \frac{F_0 (\Delta x)^3}{EI} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \\ -3 \end{bmatrix} .$$

Izraz za reakciju u zglobnom osloncu:



$$\sum F_z = 0 : F_i - R_i - Q_{i-\frac{1}{2}} = 0 \rightarrow R_i = F_i - Q_{i-\frac{1}{2}} .$$

$$Q_{i-\frac{1}{2}} = \left.\frac{\mathrm{d}M}{\mathrm{d}x}\right|_{i-\frac{1}{2}} ; \left.\frac{\mathrm{d}M}{\mathrm{d}x}\right|_{i-\frac{1}{2}} \approx \frac{M_i - M_{i-1}}{\Delta x} ; M_i = 0, M_{i-1} \approx -EI \frac{w_i - 2w_{i-1} + w_{i-2}}{(\Delta x)^2} .$$

$$Q_{i-\frac{1}{2}} \approx \frac{0 - EI \dfrac{-w_i + 2w_{i-1} - w_{i-2}}{(\Delta x)^2}}{\Delta x} = EI \frac{w_i - 2w_{i-1} + w_{i-2}}{(\Delta x)^3} .$$

$$R_i = F_i - \frac{w_i - 2w_{i-1} + w_{i-2}}{(\Delta x)^3} = F_i + EI \frac{2w_5}{(\Delta x)^3} .$$