Alen Cukrov ✉
Ivanka Boras
Josip Bašić

https://doi.org/10.21278/TOF.493079625
ISSN 1333-1124
eISSN 1849-1391

A REVIEW OF THE APPLICATION OF SMOOTHED-PARTICLE HYDRODYNAMICS IN THE COMPUTATION OF BOILING FLOWS

Summary

The paper presents a review of the research on the application of smoothed-particle hydrodynamics in the modelling of boiling flows due to the importance of such phenomena for power engineering and the lack of this type of reviews in the literature published so far. The associated high-density ratio between two fluids in a flow, with a sharp interface between the phases preserved, has been identified as a critical problem in the development of computational algorithms. This study is motivated by the recent trends in industry that are related to the application of smoothed-particle hydrodynamics. In addition, the research conducted in the field of boiling and, to some point, evaporation phenomena, is also presented since there is not much research work published in the field of boiling flow modelling using the smoothed-particle hydrodynamics method. Although some oscillations in the interface position estimation were observed, it was found that high-density ratio in conjunction with boiling flow could be efficiently modelled using hybrid methods. In particular, this refers to the application of the smoothed-particle hydrodynamics-moving particle semi-implicit method. Still, more work has to be done to establish a straightforward procedure that could be applied by engineering practitioners in research and development teams in the industry on an everyday basis.

Key words: *smoothed-particle hydrodynamics; boiling flow; phase change; sharp interface*

1. Introduction

The recent advances in computational resources may allow the application of Lagrangian particle-based discretisation techniques to solve large-scale industrial problems. For example, these techniques have been recently applied in the area of the automotive industry, an industrial sector that nowadays receives significant attention from the public. In addition, recent developments in the field of automotive engineering are followed by the application of the particle-based smoothed-particle hydrodynamics (SPH) modelling approach. Thus, Klos *et al.* [1] used the SHP to model the conjugate heat transfer in the lubrication system's complex geometry in the fluid domain and coupled it with commercial software to solve the heat conduction partial differential equation in the solid part. Chen *et al.* [2] adopted a vice-versa approach in the sense that the solid is modelled using SPH, while the finite-difference method (FDM) is used in the flow domain to solve the fluid-structure interaction (FSI) problem The authors used this approach due to great coupling possibilities between the methods. The treatises on solid mechanics analysis done using SPH are outside the scope of this paper and will not be discussed further here. In the area of free surface flow, the efficiency of the SHP method in the estimation of vehicle performance during driving through water was investigated by Posch [3], while the ship flooding problem is efficiently modelled in Hashimoto *et al*. [4]. The application of the method in the field of multiphase flow electrohydrodynamics was dealt with in Almasi *et al*. [5]. Please note that the aforementioned literature does not cover the topic of phase change, which is the primary focus of the present paper, nor the related topics that may stem from fields other than pure multiphase flow.

On the other hand, the research and development in the area of power engineering, the area associated with high heat transfer rates that lead to a boiling phase change, still relies on the Eulerian, mesh-based approach due to its natural handling of fluxes through computational cells, similar to the flow behaviour in an open thermodynamics system. A study done by Sato and Ničeno [6] is a reference worth mentioning with regard to the estimation of boiling characteristics, starting from nucleate boiling to film boiling via the critical heat flux. From a macroscopic point of view, one may point out the rod bundle analysis in Lo and Osman [7]. In the area of nuclear engineering, i.e., quenching associated with higher heat fluxes, one may mention the studies done by Cukrov *et al*. [8,9], with details on film boiling. In the studies [10,11,12] by Cukrov *et al*., a two-fluid VOF approach is applied in solving the film boiling phase during the immersion quenching of Inconel 600 alloy. Since the Eulerian boiling models are at present more mature than the ones in the SPH framework, we shall, where appropriate, refer to these approaches through the text with appropriate references.

The advantage of the SPH method over mesh-based methods is the easily determined interface position from the particle location without the need for interface reconstruction algorithms, as noted in Pozorski and Olejnik [13]. Thus, the application of the well-established geometrical reconstruction algorithm proposed by Youngs [14] is enabled within the framework of the SPH modelling. The modelling of phase-change phenomena is outlined as one of the challenges for future developments of the SPH method in Vacondio *et al*. [15]. The reviews on the droplet evaporation models are well presented in a recent paper by Sigalotti and Vargas [16]. The reviews [13], [15], and [16] provide a lot of useful data for our study; therefore, we shall return to these papers later in the text.

Due to the industrial significance of the boiling process and a recent trend towards the application of SPH in industry, the aim of this study is to investigate the possibilities of modelling the sharp interface and high-density ratio in boiling flows using SPH. The focus of the present paper is primarily on the physics modelled using SPH techniques. Therefore, we shall primarily address the studied test cases that encompass evaporation or boiling phenomena using a certain approach. Details regarding computational algorithms will be given where necessary.

The paper is organised as follows. The second chapter brings brief insights into the SPH modelling approach, boiling theory, and computation of boiling flow. To support our knowledge, adequate references related to the work in the area of topics discussed within this section will be provided. A review of the published literature on the modelling of boiling using SPH is the content of the third chapter, while the discussion on the available models is given in the fourth chapter. Thus, different fields are briefly addressed, providing a reference to the interested reader for further exploration in the field. The paper ends with conclusions.

2. Theory and modelling

First, the SPH method is briefly introduced, based on the information given in the papers by Sigalotti and Vargas [16], Chen *et al*. [17], and Pozorski and Olejnik [13]. Please see the recent study by Le Touzė and Colagrossi [18] for a comprehensive review of multiphase flow modelling with free surface using the SPH modelling approach. The theory of boiling in subsection 2.1 is based on the textbook by Galović [19] and the handbook by Liščić *et al.* [20]. Finally, the chapter ends with a brief description of key issues in the modelling of boiling flow.

2.1 A brief introduction to SPH

In the SPH method, the domain is discretised into a set of particles, each carrying physical properties, such as mass, velocity, density, and internal energy. These particles interact with each other within a finite spherical region defined by a smoothing kernel function. The kernel interpolation is the core concept of SPH, enabling the approximation of field variables and their gradients by weighting contributions from neighbouring particles [12,15]. Traditional computational fluid dynamics (CFD) methods are based on solving partial differential equations (PDEs) using finite difference, finite volume, or finite element techniques, which directly compute spatial derivatives on a grid. In contrast, SPH is based on integral approximations rather than direct differentiation. The value of a field variable at a point is approximated by integrating (i.e., summing) weights over neighbouring particles, while gradients are obtained by applying the gradient operator to the kernel itself. Therefore, the accuracy and stability of the interpolation depend on the choice of the kernel function and the distribution of particles; see, for example, Le Touzė and Colagrossi [18] for a brief introduction. As the particle influence is confined to a local neighbourhood in the finite radius of the kernel, it significantly simplifies the implementation of complex geometries and moving boundaries. However, this local nature introduces challenges at domain boundaries, where particles may lack a sufficient number of neighbours for accurate interpolation. To address this boundary truncation issue, techniques such as the insertion of mirror particles (which mimic the behaviour of fluid particles outside the domain) and dummy particles (artificial particles placed to complete the kernel support) are commonly employed. Among other problems, with a full list given in the recent work by Yoo [21], the method suffers from difficulties in turbulence handling and high computational cost. Turbulence handling within SPH is reported in Makris *et al.* [22], where the authors solve coastal free surface flow. On the other hand, the two-fluid SPH proposed in Deng *et al.* [23] may be regarded as a promising tool in saving computational resources. The computational cost of SPH is generally higher than in the case of standard Eulerian methods, as noted in O'Connor *et al.* [24], due to a larger stencil that is taken into consideration.

2.2 Theory of boiling

Let us consider a pool of water heated at the bottom surface, where, after a sufficient amount of heat is transferred to a layer in the vicinity of the heated surface (especially in the case with rough surfaces), a vapor bubble will appear. Once generated, the bubble is supplied with the heat from the bulk liquid since the liquid is in a metastable superheated state; consequently, the bubble evaporates at the interface between the vapor and liquid. Then, the vapour bubble grows and, at a certain time instance, detaches from the nucleation site with the frequency of detachment that depends on the heat load imposed on the bottom surface. Due to the presence of a gravity field, the bubble is advected to the free surface of the pool by buoyancy. For a detailed analysis of forces acting on a bubble in the case of pool nucleate boiling, please consult the study by Bucci *et al.* [25]. To transfer the heat from the liquid to vapour some, at least small, temperature differences should exist. Thus, the temperatures of the vapour and the liquid phase are different.

If we consider boiling in a pool, as it was done in the experiment by Nukiyama (1934), different boiling modes will occur under different heat flux inputs. The description of Nukiyama's experiment is available in Galović [19] and will be briefly reinterpreted in what follows. An electric wire is placed in a liquid pool at atmospheric pressure. The wire is made of a chromium-nickel alloy that has a melting point at 1500 K. The heat flow rate is obtained by applying proper voltage and electric current. The information about the surface temperature is known from the temperature-dependent electric resistance of the wire. Thus, at the beginning of the process, we shall observe the convective boiling phase. A further increase in heat input leads to the vapour bubble generation (nucleate boiling) characterised by high heat outputs at moderate temperature differences, a favourable boiling mode in power engineering. After a critical heat flux (CHF) point has been reached, the heat transfer surface cannot be wetted by the liquid anymore, and the vapour blanket surrounds the hot object (film boiling). This is an unfavourable boiling mode in power engineering since it may cause the deterioration of the heat transfer surface. Conversely, this boiling mode is very common in metallurgy, where due to extremely high heat input, the film boiling takes place at the beginning of the process. After the surface temperature reaches the minimum point, that is, the Leidenfrost temperature, also known as the re-wetting temperature, the vapour film collapses and an immediate transition to nucleate boiling takes place. The transition boiling region, however, may be achieved in the case of the constant temperature of a solid surface instead of a controlled heat flux; thus, the intermittency between the nucleate and the film boiling mode is present in the vicinity of the heat transfer surface.

2.3 Computation of boiling flow

In the area of boiling computation, Sato and Ničeno [26], in their seminal work, proposed a new approach to the sharp interface modelling. In the proposed mass transfer model, the mass transfer at the interface is directly estimated from the heat flux balance at the interface. In the calculation of the temperature gradient at the interface required for the estimation of the heat flux at each side of the interface, the evaluation of distance between the interface and the interface's neighbouring cell centre is carried out in the orthogonal directions, as shown in the study by Perez-Raya and Kandlikar [27]. Their model was inspired by the work of Sato and Ničeno [26]; the model uses three probes to estimate the distance from the nearest cell's centre to the interface. In both studies, the one-dimensional Stefan problem was used as a starting point in the evaluation of the mass transfer performance of the proposed computational model. Hence, in this study, the primary focus of each approach is its capability of solving the Stefan problem. A comprehensive review of the progress in the area of boiling modelling using Eulerian approaches is available in Kharangate and Mudawar [28]. In the area of two-fluid modelling, effective solutions to the Stefan problem are reported in Fleau [29]. The presented solutions are achieved using the large interface model (LIM) and large bubble model (LBM), while the two-fluid volume of fluid (VOF) model is applied in Cukrov *et al*. [30] to this end. For a review on the work related to two-fluid modelling with identifiable interfaces, see Mer *et al*. [31].

The fact is that the available literature on boiling within the framework of SPH is relatively scarce, and the main paper on boiling modelling using the sharp interface approach together with the high-density ratio is the work by Duan *et al*. [32]. Therefore, in this review, a reference to the study by Yang and Kong [33] is made for consideration of small density variations, and later on also to some studies dealing with evaporation. In the study by Abolmalli *et al*. [34], the Schrage mass transfer model is used to model the thin-film evaporation process. Since the Schrage model is one of the three well-established mass transfer models for the boiling flow (the other two are the energy jump and the Lee model), the consideration of evaporating flows at this point is justified. Additional justification is found in the cases studied in Bureš and Sato [35], where the Stefan and sucking problems were regarded as evaporation cases. Both mass transfer modes are dealt with in a recent work by Jouhara and Robinson [36]. To facilitate the understanding of the basic flow features associated with static droplet evaporation, the interested reader is directed to the study by Mialhe *et al*. [37], in which a direct numerical simulation (DNS) of turbulence is applied to this end. The recent advances in the field of boiling flow computation include the application of the edge-based interface tracking (EBIT) [38], geometric VOF [39] and discontinuous Galerkin (DG) [40] methods.

3. Literature review

In this chapter, a literature review on boiling and evaporation models proposed in the period from 2013 to 2024 is presented. The study done by Szewc [41] is selected as a starting point, in which the necessity for the incorporation of boiling models in SPH is pointed out. The review ends with the recent studies done by Amrofel *et al*. [42] (evaporation) and Han *et al*. [43] (boiling), which offer solutions to this end. Although the present review covers the published work in the aforementioned time frame, some publications that deal with the modelling of boiling flow are not further examined herein because they have already been discussed in the study by Duan *et al*. [32]. This study is considered as a reference in this present paper. The main focus of this review is the standard test case for the evaluation of boiling models that use a sharp interface approach, i.e., the Stefan problem. Due to the lack of such models, other cases of interest involve static droplet evaporation, droplet impact on a heated surface, and related phenomena aimed at verifying or validating the computational models in the area of thermally induced phase change.

The development of the SPH method for the computation of flows with interfaces was presented in Szewc [41] in 2013. In the code development process, the author addressed standard test cases, such as isothermal and non-isothermal cavity flows, the Rayleigh-Taylor instability, and the static droplet case presented in Brackbill *et al*. [44] (hereinafter abbreviated as Brackbill's case, according to [26]). Buoyancy-driven flows are also examined by the involvement of the Boussinesq hypothesis that refers to the linear dependence of density on the fluid temperature in the buoyancy term of the momentum equation, with density being kept constant in other terms [45]. The future work assessments, identified by Szewc [41], that should follow in the development of SPH, are related to the phase change computations, outlining thereby the boiling flow as a critical issue due to the high-density difference that occurs in vapor-liquid phase change phenomena. In addition, Szewc [41] emphasises the boiling phenomenon as a challenging topic in the Eulerian approach too. The author also points out some remedies in tackling this issue within the framework of the SPH modelling, namely adding new fluid particles (vapour) and the consideration of the variable mass and volume of particles. In his study [46] from 2017, as noted in the introductory part in Liu *et al*. [47], Szewc covered the topic of terrain deformation, for which discontinuum models can be used.

The problem of particle and jet evaporation is considered using the smoothed discrete particle hydrodynamics (SDPH) in the study by Fu-Zhen *et al*. [48]. The SPH is combined with a finite volume method (FVM), which is employed to simulate the flow of the carrier phase (bulk fluid). Additional information concerning the coupling mechanism is provided in [49]. The computational results are compared with those derived from the discrete particle model method (DPM), demonstrating a close match between the solutions. For recent studies on the coupling of FVM and SPH, the reader is referred to the work of Werdelmann [50].

The idea of fast SPH computation of boiling flow by the splitting of particles into parent and children particles is presented in Ren *et al*. [51]. The authors presented the boiling case in a vertical column. The efficiency of the computational approach is guaranteed by considering only gas particles rather than both phases. The interaction between gas and liquid particles is accomplished by imposing forces on the discrete gas particles. The work by Ren *et al*. [51], however, is directed to the area of computer visualisation that, although qualitative in nature, also considers standard interface capturing techniques (such as the level-set method, noted in Wojtan *et al.* [52]) used in the numerical modelling of multiphase flow; see, e.g., Heiss-Synak *et al*. [53] for recent progress in the field related to bubble modelling, and Xu [54] for a discussion on the application of SPH to this end.

In the area of modelling the multiphase flow with evaporation, a computational model proposed by Yang and Kong [33] is referred to as a convenient approach for evaporating flows with small density variations in the discussion part in Duan *et al*. [32]. Yang and Kong's method uses particle splitting and merging to model the mass transfer that is to appear in the evaporation process. Thus, after a certain particle mass criterion is met, merging or splitting of a particle occurs, as shown in Fig. 1 on the example of two particles, *a* and *b*. Furthermore, the interface between the phases is rather diffusive than sharp. The mass transfer is modelled using the species concentration in the vapour phase and it is expressed as:

$$\dot{m} = \frac{V\,\nabla\cdot(\rho D \nabla Y)}{1-Y} \tag{1}$$

where V is the volume; ρ is the fluid density; Y is the vapour mass fraction. Written in the discrete form for the gas particle, Eq. (1) reads:

$$\frac{\mathrm{d}m_g}{\mathrm{d}t} = \sum_l \dot{m}_{gl} = \sum_l \frac{2 m_g m_l D_g (\vec{r}_g - \vec{r}_l)\cdot\nabla_g W_{gl}}{\rho_l (r_{gl}^2 + \eta)(1 - Y_g)} (Y_g - Y_l) \tag{2}$$

where m_g is the mass of a gas particle; $\dot{m}_{gl}$ is the mass transfer rate from liquid to gas particle; m_l is the liquid particle mass; D_g is the mass diffusion of a gas particle; $\vec{r}_g$ and $\vec{r}_l$ are the position vectors of gas and liquid particles, respectively; $\nabla_g W_{gl}$ denotes the gradient of kernel function W_{gl} with respect to the position coordinates of a gas particle; ρ_l is the liquid particle density; r_{gl}^2 is the squared distance between the gas and the liquid particle; η is the term required for preventing singularity if the particles are too close to each other; Y_g is the vapour mass fraction in the gas particle; Y_l is the vapour mass fraction in the liquid particle that is treated as a gas particle at the interface, as noted in [55]. An identical approach is used for the liquid phase, just with the opposite sign. The authors show the method performance by solving the one-dimensional Stefan problem (liquid evaporation and a vertically placed liquid column), static droplet evaporation, and drop impact on a heated surface. The latter is pointed out as the case where the evaporation model has found its widest application, according to the review by Sigalotti and Vargas [16].

The modelling approach proposed in Yang and Kong [33] in 2017 was used in the qualitative analysis of flow behaviour under the impact of a drop on a surface with a temperature that is above the Leidenfrost temperature in Yang *et al*. [56] in 2018. A flow map was designed by the authors according to the findings from numerical simulations, relating the Weber number and the wall temperature together with the presence or absence of the Leidenfrost phenomenon in the flow. Furthermore, in a recent study by Ahamed [55], that evaporation model was used in the finite element method analysis of thermal stresses due to the drop impact on a solid wall.

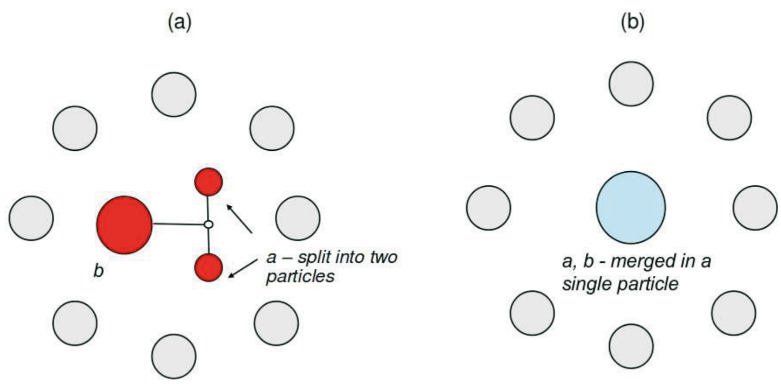

Fig. 1 Particle (a) splitting and (b) merging in the approach presented in 2017 by Yang and Kong [33].

Additionally, in 2019, Yang and Kong [57] offered an upgrade in a multi-resolution model featuring adaptivity in a different investigation. The effectiveness of the method in addressing isothermal multiphase flow test cases was successfully illustrated through the resolution of intricate drop impacts on both wet and dry surfaces, drop impacts in a deep pool, the water entry of a horizontal cylinder, and the breaching of a dam; the latter serves as a classic validation case for interface tracking.

A simulation of the boiling phenomenon on hydrophobic and hydrophilic surfaces using the SPH method is presented in Xiong *et al*. [58]. The authors used the van der Waals model and caloric equation to close the governing equation set, while benchmarking was carried out using three different cases. In the application part, a partially and totally heated surface was considered, and the occurrence of the critical heat flux was examined. Furthermore, the effect of wettability on the boiling heat transfer was also verified by the authors. This approach is regarded as a diffuse interface approach in the recent work by Zhuang *et al*. [59].

A numerical simulation of water-liquid evaporation using the commercial SPH code was conducted in Wickert and Prokop [60]. In their study, the boiling model known as the Lee model was addressed, although not used, in the context of the evaporation phenomena. Although there are discrepancies between the SHP model results and the experimental results, the SHP model has shown its ability to account for wetting processes.

The sharp-interface treatment in conjunction with the high-density ratio handling in the simulation of boiling phenomena using the weakly compressible SPH (WCSPH) method coupled with the moving particle semi-implicit (MPS) method in the incompressible-compressible SPH and MPS (IC-SPH-MPS) approach was proposed in Duan *et al*. [32]. In that approach, the SPH algorithm is applied in the vapour phase due to its stable computation of newly added particles in the mass transfer process, while the MPS approach is used in the liquid phase. The liquid phase acts as a wall boundary for the vapour phase, providing the SPH algorithm with velocity and zero-gradient pressure (the von Neumann boundary condition). On the other hand, the vapour phase, which is handled using the SPH method, functions as a boundary for the specification of pressure (the Dirichlet boundary condition) and imposes the no-slip velocity constraint on the liquid phase. The applied mass transfer model reads:

$$\Delta m_i = \frac{\rho_l C_{p,l}(T_i - T\prime)V_0}{r_0} \tag{3}$$

where ρ_l is the liquid density; $C_{\mathrm{p,l}}$ is the specific heat capacity of liquid; T_i is the temperature of the liquid particle; T' is the saturation temperature; V_0 is the particle volume; and r_0 is the specific heat of vaporization. The particles at the interface are held at saturation temperature. A comparable method, involving the "locking" of interface cells to interface temperature (equilibrium phase-change model), was presented in Pan *et al*. [61], where the Eulerian-VOF approach was employed to describe boiling. Furthermore, the mass transfer model used in Duan et al. [32] is, to some extent, similar to the aforementioned one in Pan *et al*. [61] by the involvement of sensible heat in the numerator of the mass transfer term; the difference between the two models is that the heat transfer model used in [32] refers only to the liquid side of the interface. In that model, the mass is added to the liquid interface particles, but only temporarily. If the liquid particle's mass exceeds the gas particle's mass and the particle number density (PND) criterion is met, the new cell is generated and added to a gas phase towards the liquid interface particle. Consequently, mass transfer is represented by the introduction of vapour particles preceding the liquid interface particles, which are classified as boiling based on a surface tension algorithm that meets the threshold angle condition. The SPH method is utilised for vapour particles because of its effective management of circumstances involving the introduction of additional particles into the flow. Furthermore, the surface tension is computed exclusively for the liquid particles to maintain computational stability, i.e., to avoid severe mixing of particles that is reported by the authors in the case when both phases are involved in surface tension computation. The steps of the algorithm are shown in Fig. 2.

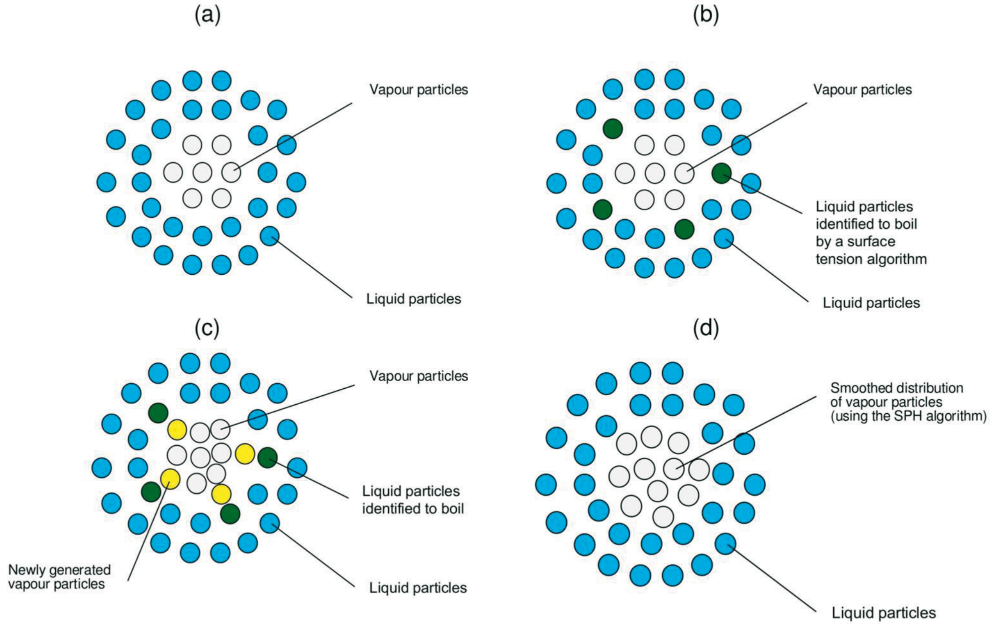

Fig. 2 Steps of the boiling computation algorithm in the heat transfer model presented in Duan *et al*. [32]. (a) after the temperature is calculated, some liquid particles are identified as boiling ones, as the ones that have the temperature higher than the saturation temperature; (b) inserted vapour particles; (c) smoothed vapour particles; (d) expansion of liquid particles due to high vapour pressure.

In the calculation procedure, the vapour particles are handled firstly by using the SPH approach with a small time-step size. Despite employing various computational methods for the estimation of liquid and vapour phases, the temperature field is subsequently computed concurrently for both phases. After this step, liquid particles are solved by using the MPS approach. Particles designated for boiling are identified via the application of the MPS technique. The liquid expansion is the result of the elevated pressure of vapour particles and is calculated using the MPS method. The authors effectively modelled the Stefan problem, the suction problem, and the film boiling on a horizontal plate, for which Berenson [62] presented an equation for calculating the Nusselt number and, consequently, the heat transfer coefficient. The main findings of research conducted by Duan *et al*. [32] may be summarised as follows: (i) the application of the lowest-rank search algorithm in the pressure calculation due to the high speed of sound that was necessary for modelling incompressible flow; (ii) the surface tension is calculated only in the liquid phase (free surface-based approach) in order to avoid severe particle mixing that may arise if both phases are involved in the surface tension calculation (interface-based approach); (iii) the high-density ratio yields significant volume expansion that can be successfully modelled by the insertion of particles towards the liquid phases by means of the explicit conservative SPH method for the gaseous phase.

An implicit treatment of the liquid-vapour phase change, in terms of evaporation-induced recoil pressure and heat losses, was applied in the additive manufacturing study by Meier *et al*. [63]. The authors used the WCSPH approach to study the influence of the laser beam on the material, considering thereby a wide variety of mechanical and thermal fluxes at the interface between the phases.

The high-density ratios that occur during boiling simulations have been handled in the software used by Dong *et al*. [64]. The authors, however, modelled a liquid-solid phase change process for a wide set of test cases, involving the one-dimensional Stefan problem. Other tested cases include Brackbill's case, the rising bubble and the rising bubble coalescence. The examples of application are directed to model the complex flow behaviour after the entry of the solid material into the liquid and the corresponding solid-liquid phase change. The importance of handling the high-density ratio within boiling simulation has been recently underlined by Zhang *et al*. [65].

The Cahn-Hilliard model for separated flows was used in an interface-capturing phase-field method for the computation of boiling flows in Wang, Z. *et al*. [66] and was addressed in the recently proposed energy-stable solution for incompressible flow using the SPH method in Feng *et al*. [67]. Hence, one may reduce the need for standard techniques for incompressible flow modelling within the framework of SPH, WCSPH, and incompressible smoothed-particle hydrodynamics (ISPH) approaches, raising the possibility for the incorporation of higher time steps in the simulations. The model is, among other cases, verified on the aforementioned Brackbill's case and shows great potential. Since the Cahn-Hilliard are the actual equations used in the modelling of boiling flow, we should also mention the recent work on the three-phase meshless modelling by Ghoneim [68] and the references therein. The application of the Cahn-Hilliard model is, on the other hand, a well-established method in the area of phase-field based topology optimisation [69], as noted in the recent review by Wang, Y., *et al*. [70].

This study concerns the boiling phenomenon, wherein the phase change process is triggered by a temperature gradient; however, it is also important to mention the cavitation phenomenon, in which the phase change process occurs owing to a pressure decrease below the saturation threshold. The industrial standard for the modelling of the mass transfer in the case of cavitation is the application of the Schnerr-Sauer model, as noted in Perić [71]; this model was applied in the studies by Savio *et al*. [72] and Romani *et al*. [73]. The mathematical model that is applicable to this end was proposed in Aganin and Davletshin [74]; their model addresses the bubble-bubble interaction. In the modelling of cavitation using the SPH method, we may point out the studies by Lyu *et al*. [75] and Kalateh *et al*. [76], while the application of the method in the computation of cavitation erosion may be found in Joshi [77].

Within the context of the drying process, Amrofel *et al*. [42] proposed an evaporation-condensation mass transfer model that considers the capillary and the Kelvin effect. An in-house SPH computational code is verified, from the viewpoint of mass transfer, on the evaporation of a liquid with and without the presence of the capillary or the Kelvin effect. The final case studied by the authors is related to the model application in the numerical simulation of flow phenomena in a connected pore network, representing a real case.

The application of SPH in a distillation process, an important feature of process engineering, was reported in the study by Cortez-Gonzáles *et al*. [78]. For more information regarding the optimisation in the distillation process, with a focus on the batch distillation, the interested reader is directed to the study done by Zadravec *et al*. [79].

The particle splitting and merging technique in a combined MPS-SPH approach has been recently used in Han *et al*. [43]. The authors solved standard multiphase flow test cases, such as the dam break, the rising bubble, the Stefan problem, and the sucking problem. Since the consideration of the Stefan and the sucking problem within the framework of SPH was already discussed in Duan *et al*. [32], we may underline the connections with other investigators on the topics of the other two standard test cases used in the evaluation of multiphase flow models. Thus, the dam break case was also considered in Calderon-Sanchez *et al*. [80], while the rising bubbles were addressed in the recent work by Fang *et al*. [81].

4. Discussion

From the presented work in the area of phase-change modelling using SPH, one may point out three main approaches with a primary focus on the boiling phenomenon. Two of them, dealt with in [33, 42], refer to the modelling of the evaporation process by solving the vapour phase diffusion equation. The third method, dealt with in [32], is used for simulating the boiling process by using the high-density ratio and a sharp interface between the phases. The high-density ratio is also present in the case of evaporation, but the mass transfer is induced by the gradient of the species mass fraction. Please check Boniou's study [82] for further discussion on this topic. A study carried out by Cipriano *et al*. [39] encompasses both the temperature gradient (boiling) and the species concentration gradient (evaporation) induced by phase change processes; the study is done using a geometric VOF modelling framework. Furthermore, based on the classification provided in Duan *et al*. [32], methods are divided into those that are suitable for low-density ratios, such as the method proposed in [33], and the ones that are more suitable for the cases of high-density ratios, such as in the case considered in [32].

In the case of diffusive interface handling, SPH used as the sole method has proven its accuracy in solving phase-change phenomena in a vapour-liquid system. However, when it comes to the sharp interface treatment, a conjunction with another method is necessary. Thus, the application of SDPH, in which SPH is used with FVM, has been confirmed as an efficient tool in the modelling of particle jet evaporation.

Although fully Lagrangian, the problem of spurious velocities is also present in the framework of the SPH method, as noted in Pozorski and Olejnik [13]. The problem is about the existence of an unphysical velocity field in the vicinity of an interface; it is caused by the disbalance between the pressure and surface tension forces. This can be easily deduced from the Laplacian solution of pressure jump in the case of a static droplet in a zero-gravity environment, where the pressure difference is equal to the surface tension multiplied by the curvature, i.e., $\Delta p = \sigma \varkappa$. Since the velocity field in that case should be zero, in the case of numerical simulations, one can note the existence of the unphysical velocity field known as spurious velocities, studied in the work by Gunstensten [83]. To the authors' knowledge, Gunstensten's study is one of the first studies on the spurious velocities. In that study, the author concludes that high capillary numbers yield small spurious velocities and vice versa, expressing the relationship as $v_{\max} \propto Ca^{-1}$. This relationship is also confirmed in the research done by Zahedi *et al*. [84], referring thereby also to the work by Lafaurie *et al*. [85]. The approach suggested in Mathieu [86] is noted as a promising approach to handling spurious velocities in the work by Toutant *et al*. [87].

Both the diffusive and sharp interface representations utilise the one-dimensional Stefan problem for model verification; nevertheless, obvious distinctions between the two techniques can be seen. In the context of evaporative model verification, the Stefan problem pertains to the evaporation of a liquid in a vertical orientation, with model accuracy assessed through the measurement of vapour fraction at the interface. Conversely, in the sharp interface approach, the position of the interface is explicitly tracked.

Moreover, a combination with MPS has advantages, as proven by Duan *et al*. [3], for the precise interface modelling in classic film boiling scenarios (the Stefan problem, suction issue, and horizontal film boiling). Conversely, previous findings indicated that the exclusive application of MPS [88] or the utilisation of MPS together with meshless advection employing the flow-directional local-grid approach [89] may successfully track bubble dynamics in the boiling regime explicitly. This approach belongs to arbitrary Lagrangian-Eulerian (ALE) formulations in which the interface is tracked explicitly, and hence, there are no spurious velocities in the flow. The application of the ALE method in the computation of boiling flows is reported, for example, in Gros [90]. Other usages of ALE in conjunction with SPH are related to molten corium spreading in nuclear engineering, presented in Yoo [21], and the SPH-ALE tracking of shock-wave impact on the structures, reported in Messahel [91]. The boundary deformation that requires the application of remeshing associated with the ALE approach (for remeshing within ALE please check the studies done by Olivier [92] and Barral [93]) is remedied in the coupled

Eulerian-Lagrangian (CEL) approach, as indicated in Ducobu *et al*. [94]. This approach, presented in the paper by Noh [95] in 1993, has been recently applied in solving the debris problem in [96], where SPH could also be used; some of the applications, according to [96], one may find in the recent papers by Shi *et al*. [97] and Ng *et al*. [98].

The studies on boiling by Xiong *et al*. [58] and evaporation by Wickert and Prokop [60] deal with the capability of the SPH method for solving the wetting phenomenon. This is especially important for studying the boiling phenomenon in the quenching process since it influences the temperature distribution in a solid material [99]. For example, the wetting effects are included in the quenching simulation via the heat transfer coefficient, as noted in Felde and Shi [100]. Basic research in the area of surface tension and wetting phenomena may be found in [101], while a recent development in the field may be found in the paper by Olejnik and Pozorski [102]. The problem of triple contact line that may be associated with the wetting phenomenon has been thoroughly tackled from a mathematical point of view in the recent work by Eyal *et al*. [103]. The dynamics of the contact line in the case of boiling was studied in Mathieu [86] and Nichita [104].

The solution to the Stefan problem and the horizontal film boiling within the scope of hybrid models is presented in Liu [105], where a coupled CIP/MM FV FVP method (constrained interpolation profile/multi-moment) is used.

The energy-stable scheme incorporated in Feng *et al*. [67] has been found as a promising approach to solving the tensile instability associated with the application of SPH. Further investigations into the application of the energy-stable scheme within SPH can be found in the recent work by Zhu [106] and Zhu *et al*. [107]. Thus, in [106], dealing with the high-density ratio is noted as one of the features of this approach.

The solution to the Stefan and the sucking problem obtained in Duan *et al*. [32] has shown oscillations, compared to the analytical solution. The cause of these oscillations, as claimed by the authors, is the addition of new particles to the phase change process. Therefore, the oscillations are higher in the sucking problem due to, as noted by the authors, more violent boiling than in the Stefan problem, where boiling is considered to be mild. It is interesting to note that the oscillations were also observed in the solidification case (the Stefan problem) in Lyu *et al*. [108], where a coupled VOF-IBM (immersed boundary method) was used. Moreover, the oscillations of the interface position were also observed in another application of SHP, i.e., in the case of melting presented in Dong *et al*. [64].

5. Conclusions

The paper considers the advancements in boiling and evaporation modelling in the period from 2013 to 2024 and deals with related challenges such as high-density ratios and various coupling methods. The following conclusions are drawn from the reviewed literature:

- The efficient handling of boiling phenomena through a sharp interface approach benefits from the utilisation of hybrid models, specifically the integration between smoothed-particle hydrodynamics (SPH) and moving particle semi-implicit (MPS) methods.
- The transient oscillations across interface positions resulting from the creation of new particles in the incompressible-compressible SPH-MPS method have also been documented in the melting SPH and solidification volume of fluid (VOF)-immersed boundary method (IBM) solutions.
- The high-density ratio, crucial for boiling simulation, must be effectively managed to maintain a sharp interface, whereas a diffusive technique may be employed for low-density ratios (evaporation).

- The wetting phenomena, crucial for modelling the boiling flow related to the quenching process, has been well modelled using the SPH approach.
- The adoption of energy-stable formulation in the SPH modelling has proven to be an effective method for achieving incompressible flow simulations with increased time steps. Moreover, it was determined to be an effective method for addressing high-density ratios.

In conclusion, more work should be done within the framework of the SPH modelling approach in order to make it capable of handling sharp interface high-density boiling flows. This is not solely applicable to Lagrangian methods; the same holds true for effective industrial-scale computations within the Eulerian framework. The two-fluid SPH method described by Monaghan and Kocharyan [109], as referenced in [13], is considered a promising strategy for decreasing computing expenses.

Dedication

This paper is dedicated to the 75th birthday of Professor Emeritus Antun Galović (27th April 2025) and Boris Halasz, PhD, Professor in retirement (1st September 2025).

Acknowledgement

The first author would like to thank Dr. Martina Odeljan for preparing Professor Emeritus Antun Galović's CV. The literature suggestion on cavitation from Klara Pešić, Laboratory for Simulation and Modelling, Paul Scherrer Institute, Switzerland, is gratefully acknowledged. The discussion on the mass transfer during the evaporation process with Mr. Nikola Borovnik, MS and Dr. Danijel Zadravec is also gratefully acknowledged.

CV of Professor Emeritus Antun Galović, PhD

Antun Galović was born on 27th April 1950 in Sikirevci, where he completed elementary school. He attended a secondary technical school specialising in mechanical engineering in Bosanski Šamac. In 1969, he graduated and enrolled at the Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb. He completed his studies in mechanical engineering, specialising in thermal engineering, in June 1974.

As of 1st October, 1974, he was employed at the Faculty of Mechanical Engineering and Naval Architecture in Zagreb as an assistant teaching the courses Heat Science I and Heat Science II. He obtained his master's degree in 1979 and earned his doctorate in 1985, defending a dissertation entitled "*Heat Exchange Between a Fluidized Bed and an Immersed Heat Exchange Surface*". He was appointed assistant professor in 1986, associate professor in March 1991, full professor in February 1998, and full professor with tenure in 2003. He was granted the title of professor emeritus in 2021.

From 1992, he was appointed Head of the Chair of Heat and Thermal Devices, and from 1996 until his retirement, he was Head of the Chair of Technical Thermodynamics within the newly established Department of Thermodynamics and Thermal and Process Engineering.

He was elected as an associate member of the Croatian Academy of Technical Sciences in 1998 and as a full member in 2013. Since 2006, he has been a member of the International Centre for Heat and Mass Transfer (ICHMT), the Croatian Energy Regulatory Agency (HERA), and the Croatian Section of the Combustion Institute.

Professor Galović has received numerous prestigious awards in recognition of his contributions to teaching and scientific work in the field of technical thermodynamics. He was awarded the Small Medal of the Faculty of Mechanical Engineering and Naval Architecture in 1985 and received the "*Fran Bošnjaković*" Award of the University of Zagreb in 1999. In 2004, he received the "*J. J. Strossmayer*" Award, and in 2009, the Croatian Academy of Engineering Annual Award "*Rikard Podhorsky*". He was also awarded the Great Medal of the Faculty of Mechanical Engineering and Naval Architecture in 2017, as well as the Honorary Charter of the Technical Faculty of the University of Bihać in 2018.

Professor Galović is the author of two university textbooks, *Thermodynamics I* and *Thermodynamics II*, which have been issued in multiple editions. He is also the co-author of the *Collection of Problems in Thermodynamics II* and the comprehensive volume titled *Thermal Tables*. He contributed to Volume XIII of the *Technical Encyclopedia*, covering topics such as heat transfer modes, heat exchangers, and combustion. These topics are also addressed in Volume I of the *Engineering Manual*.

The early scientific work of Professor Galović focused on the field of unsteady heat transfer. In this context, he developed a numerical procedure based on the finite element method for three-dimensional heat conduction in a plane wall with a moving heat source. The method proved applicable in the field of welding. He also applied a variational approach through the variation of a functional, which served as the basis for the model formulation. At the time, this was one of the first studies to numerically and systematically address this problem. Furthermore, Professor Galović devoted a significant part of his scientific career to heat transfer in gas-solid fluidized beds, specifically air and sand particles of varying diameters. Drawing on the contemporary literature of the time, he developed an analytical model for calculating the heat transfer coefficient between a fluidized bed and an immersed heat exchange surface. To support this, he also designed an experimental setup, which demonstrated the benefits of using a fluidized bed in heat transfer processes. He showed that there exists an optimal fluidization regime at which the maximum heat transfer coefficient is achieved. The same experimental setup was later used for another doctoral thesis, for which Professor Galović served as mentor. In that thesis, the existence of the optimal fluidization regime was experimentally confirmed.

Later, Professor Galović's scientific work focused on the energy and entropy analysis of gas turbine operation. He also developed an entropy analysis of the adiabatic mixing of three streams of different ideal gases at the same pressure and temperature. Professor Galović concentrated his further research on dimensionless entropy and exergy analysis of thermal processes, primarily in the field of heat exchangers. He also formulated cases of local entropy generation under steady-state conditions through a plane solid wall, both with and without the presence of a heat source or sink. Through his work, Professor Galović continued the relentless fight against irreversibility, first emphasised by the great Professor Bošnjaković in his visionary 1938 paper *"Kampf den Nichtumkehrbarkeiten"*.

As part of his research work, he led three scientific projects funded by the Ministry of Science of the Republic of Croatia, focusing on heat and mass transfer in fluidized beds. He also contributed to several expert studies, playing a particularly significant role as co-author of a project involving the injection of carbon dioxide into oil wells to recover residual depleted reserves.

He is the author of numerous scientific and professional papers published in national and international journals and has supervised three doctoral and five master's theses.

Professor Galović's teaching activity included lectures in the undergraduate courses Thermodynamics I, Thermodynamics II, Fundamentals of Thermodynamics A, and Thermodynamics of Materials at the Faculty of Mechanical Engineering and Naval Architecture. At the doctoral level, he prepared and taught courses such as Special Topics in Conduction and Exergy Analysis of Thermal Processes and developed models of radiative heat transfer as part of the course Heat and Mass Transfer. For almost three decades, Professor Galović delivered lectures in Thermodynamics I and Thermodynamics II at the Faculty of Mechanical Engineering in Slavonski Brod, maintaining uninterrupted teaching even during the Croatian War of Independence.

It is particularly noteworthy that Professor Galović saw his profession as a lifelong mission and devoted his entire career to students, primarily to the transmission of knowledge in the field of technical thermodynamics. His lectures were always clear, well-structured, dynamic, and inspiring, and his enthusiasm was boundless. Over the years, students respected him for his expertise, dedication, accessibility, responsibility, and the respect he showed toward them. During his career, nearly 18,000 students had the opportunity to learn from him — a number comparable to an entire city of engineers.

At his final lecture in June 2019, a full auditorium of students, alumni, and faculty staff gave him a standing ovation, honouring his decades of dedicated and successful work.

We wish Professor Emeritus Antun Galović good health and a cheerful spirit, and that his scientific curiosity continues to guide him along the yet-unexplored paths of thermodynamics.

CV of Boris Halasz, PhD, Professor in retirement

Boris Halasz was born on 1st September 1950 in Zagreb, where he completed elementary and secondary education (Classic secondary school). He enrolled in the Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, in 1969. After graduation in 1974, he was appointed at the same institution in 1975 and worked there until his retirement in 2015. In 1984, he defended his master's thesis entitled "Heat transfer in heat regenerators and the methods of their rating" and his PhD thesis entitled "Indirect evaporative cooling in a heat exchanger with one-side wetted wall" in 1992.

Professor Halasz's most recognised scientific work is related to the heat transfer analysis of heat exchangers. To this end, he proposed an approach to tackling heat transfer characteristics of different types of heat exchangers using a general mathematical model. Thus, by taking the working fluid, air moisture, and air and water energy balances in a differential equation form, one may obtain the characteristics related to the heat transfer performance of evaporative coolers, which is the most complex problem. This approach can also be adopted for the subset of evaporative coolers, i.e., cooling towers and evaporative condensers. The general mathematical model used simply eliminates the terms that do not exist in the specific heat exchanger type. The proposed solution is actually an extension of Professor Bošnjaković's idea for recuperative heat exchangers using the linearisation of the air saturation curve. One of the advantages of Professor Halasz's approach in the case of cooling towers is that the air temperature and the moisture content are treated as separate variables that allow for calculation of the percentage of evaporated water. Furthermore, using this approach, one may obtain solutions for the cross-flow cooling tower.

We should also acknowledge the work of Professor Halasz in the area of ice banks, on a project that was aimed at studying the behaviour of an ice bank formed as an ice silo. This project resulted in a full-scale mathematical model of the non-steady-state process of building and melting the ice in an ice bank. We should also include his studies on entropy analysis of a counterflow cooling tower and a crossflow heat exchanger and his study on air washers, all of them having been presented at international conferences.

From his first days at the Faculty of Mechanical Engineering and Naval Architecture until his retirement, Professor Halasz was heavily engaged in teaching at the Chair of Thermodynamics; the courses delivered included Thermodynamics I and II, and later in his career, Thermodynamics of Mixtures, Technical Drying Processes, and Introduction to Thermodynamics. To help students in their effort, he published several noteworthy books. The first one, *Collection of Problems in Thermodynamics I*, was published as early as in 1981, followed by a number of reprints. In 1987, Professor Halasz, with Professors Galović and Tadić as co-authors, published *Collection of Problems in Thermodynamics II*. A completely new *Collection of Problems in Thermodynamics I* was published in 2004, followed by *Collection of Problems for the Introduction to Thermodynamics* in 2009, and finally, in 2012, the book *Introduction to Thermodynamics*, which was written in accordance with the academic curriculum valid at the time. In addition, Professor Halasz produced several useful tools for the students, such as thermodynamic tables and the water-steam Mollier diagram, to mention just two.

In 1988 he received the Faculty Medal for teaching excellence, and in 2011 he was awarded for his online tests, together with his colleague who succeeded him when he retired.

After the retirement, he was engaged in collaboration with the Technical Museum Nikola Tesla in Zagreb on creating a collection of old calculating devices to be presented in the museum. This collaboration resulted in two exhibitions: "How our ancestors calculated" in 2017 and "From a slide rule to an electronic calculator in Croatia" in 2023.

After his retirement, Professor Halasz stayed in contact with the staff of his former Chair of Engineering Thermodynamics.

We wish Professor Halasz good health and much creativity in the times to come. We are extremely grateful for his contributions, thanks to which today we have easy tasks in our teaching and scientific work.

Chair of Engineering Thermodynamics, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb.

REFERENCES

[1] Klos, F.; Hoffmann, M; Rothe, S. Conjugate Heat Transfer Simulation mittels PreonLab [Conference presentation] **2024**, AVL Simulation Conference Germany 2024. 24.04.2024. Regensburg, Germany. https://www.avl.com/de-de/downloads-avl-simulation-conference-germany-2024

[2] Chen, J.-Y.; Feng, D.-L.; Peng, C.; Ni, R.-C.; Wu, Y.-X.; Li, T.; Song, X.-Z. A coupled FD-SPH method for shock-structure interaction and dynamic fracture propagation modeling, *Applied Mathematical Modelling*, **2024**, 134, 288-306. https://doi.org/10.1016/j.apm.2024.06.010

[3] Posch, K. Validierung von Wasserdurchfahrtsimulationen [Conference presentation] **2024**, AVL Simulation Conference Germany 2024, Regensburg, Germany. https://www.avl.com/de-de/downloads-avl-simulation-conference-germany-2024

[4] Hashimoto, H.; Grenier, N.; Sueyoshi, M.; Le Touzé, D. Comparison of MPS and SPH methods for solving forced motion ship flooding problems, *Applied Ocean Research*, 118, **2022**, 103001. https://doi.org/10.1016/j.apor.2021.103001

[5] Almasi, F.; Shadloo, M.S.; Hadjadj, A.; Ozbulut, M.; Tofighi, N.; Yildiz, M. Numerical simulations of multi-phase electro-hydrodynamics flows using a simple incompressible smoothed particle hydrodynamics method, *Computers & Mathematics with Applications*, 81, **2021**, 772-785. https://doi.org/10.1016/j.camwa.2019.10.029

[6] Sato, Y.; Niceno, B. Pool Boiling Simulation Using an Interface Tracking Method: From Nucleate Boiling to Film Boiling Regime through Critical Heat Flux. *Int. J. Heat Mass Transf.* **2018**, 125, 876–890. https://doi.org/10.1016/j.ijheatmasstransfer.2018.04.131

[7] Lo, S.; Osman, J. CFD Modeling of Boiling Flow in PSBT 5×5 Bundle, *Science and Technology of Nuclear Installations*, **2012**, 795935, 2012. https://doi.org/10.1155/2012/795935

[8] Cukrov A.; Sato Y.; Boras I.; Ničeno B. Film Boiling around a Finite Size Cylindrical Specimen—A Transient Conjugate Heat Transfer Approach. *Applied Sciences*. **2023**; 13(16):9144. https://doi.org/10.3390/app13169144

[9] Cukrov A.; Sato Y.; Landek D.; Hannoschöck N.; Boras I.; Ničeno B. Determination of Heat Transfer Coefficient in a Film Boiling Phase of an Immersion Quenching Process. *Applied Sciences*. **2025**; 15(3):1021. https://doi.org/10.3390/app15031021

[10] Cukrov A.; Landek D.; Sato Y.; Boras I.; Ničeno B. Water Entry of a Heated Axisymmetric Vertical Cylinder. *Energies*. **2023**; 16(24):7926. https://doi.org/10.3390/en16247926

[11] Cukrov A.; Landek D.; Sato Y.; Boras I.; Ničeno B. The development of the numerical method for simulation of metal material quenching. *Rud.-geol.-naft. zb.* [Internet]. **2025** Feb.11; 40(1):29-50. https://doi.org/10.17794/rgn.2025.1.3

[12] Cukrov A.; Landek D.; Sato Y.; Boras I.; Ničeno B. Numerical simulation of film boiling heat transfer in immersion quenching process using Eulerian two-fluid approach. *Case Studies in Thermal Engineering*. **2024** December; 64:105497. https://doi.org/10.1016/j.csite.2024.105497

[13] Pozorski, J.; Olejnik, M. Smoothed particle hydrodynamics modelling of multiphase flows: an overview. *Acta Mech* 235, 1685–1714, **2024**. https://doi.org/10.1007/s00707-023-03763-4

[14] Youngs, D. L. Time-dependent multi-material flow with large fluid distortion. Numerical methods for fluid dynamics. 1982.

[15] Vacondio, R.; Altomare, C.; De Leffe, M. et al. Grand challenges for Smoothed Particle Hydrodynamics numerical schemes. *Comp. Part. Mech*. 8, 575–588, **2021**. https://doi.org/10.1007/s40571-020-00354-1

[16] Sigalotti L.D.G. and Vargas C.A. (2025): Smoothed particle hydrodynamics (SPH) simulations of drop evaporation: A comprehensive overview of methods and applications. *Comput Model Eng Sci*.142(3):2281–2337. https://doi.org/10.32604/cmes.2025.060497

[17] Chen, D.; Liang, C.; Huang, W. A unified updated Lagrangian smoothed particle hydrodynamics for fluid-structure interaction problems with moving boundaries and interfaces, *Applied Mathematical Modelling*, 137, Part A, **2025**, 115661. https://doi.org/10.1016/j.apm.2024.115661

[18] Le Touzé, D.; Colagrossi, A. Smoothed particle hydrodynamics for free-surface and multiphase flows: a review. *Reports on Progress in Physics*, 88 (3), **2025**, 037001. https://dx.doi.org/10.1088/1361-6633/ada80f

[19] Galović, A. Termodinamika II, Fakultet strojarstva i brodogradnje, Sveučilište u Zagrebu, Zagreb, **2014**

[20] Liščić, B.; Tensi, H.M.; Luty, W. eds., Theory and Technology of Quenching, Springer Berlin Heidelberg, Berlin, Heidelberg, **1992**. https://doi.org/10.1007/978-3-662-01596-4

[21] Yoo, H.S. Multiphase ALE-SPH for Versatile Approaches on Highly Viscous Corium Simulation, *PhD Thesis*, Seoul National University, **2024**. https://s-space.snu.ac.kr/handle/10371/210114

[22] Makris, C.V.; Memos, C.D.; Krestenitis, Y.N. Numerical Modeling of Surf Zone Dynamics under Weakly Plunging Breakers with SPH Method. *Ocean. Model*. **2016**, 98, 12–35. https://doi.org/10.1016/j.ocemod.2015.12.001

[23] Deng, L.; Liu, Y.; Wang, W.; Ge, W.; Li, J. A two-fluid smoothed particle hydrodynamics (TF-SPH) method for gas–solid fluidization, *Chemical Engineering Science*, 99, **2013**, 89-101. https://doi.org/10.1016/j.ces.2013.05.047

[24] O'Connor, J.; Domínguez, J.M.; Rogers, B.D.; Lind, S.J.; Stansby, P.K. Eulerian incompressible smoothed particle hydrodynamics on multiple GPUs, *Computer Physics Communications*, 273, **2022**, 108263. https://doi.org/10.1016/j.cpc.2021.108263

[25] Bucci, M.; Buongiorno, J.; Bucci, M. The not-so-subtle flaws of the force balance approach to predict the departure of bubbles in boiling heat transfer. *Physics of Fluids* 1 January **2021**; 33 (1): 017110. https://doi.org/10.1063/5.0036956

[26] Sato, Y.; Ničeno, B. A Sharp-Interface Phase Change Model for a Mass-Conservative Interface Tracking Method. *J. Comput. Phys*. **2013**, 249, 127–161. https://doi.org/10.1016/j.jcp.2013.04.035

[27] Perez-Raya, I.; Kandlikar, S.G. Discretization and implementation of a sharp interface model for interfacial heat and mass transfer during bubble growth, *International Journal of Heat and Mass Transfer*. 116, **2018**, 30–49. https://doi.org/10.1016/j.ijheatmasstransfer.2017.08.106

[28] Chirag R. Kharangate, C.R.; Issam Mudawar, I. Review of computational studies on boiling and condensation, *International Journal of Heat and Mass Transfer*, 108, A, **2017**, 1164-1196. https://doi.org/10.1016/j.ijheatmasstransfer.2016.12.065

[29] Fleau. S. Multifield approach and interface locating method for two-phase flows in nuclear power plant. Fluid mechanics. *PhD Thesis*. Université Paris-Est Marne-la-Vallée, **2017**. https://theses.hal.science/tel-01578335/

[30] Cukrov, A.; Sato, Y.; Boras, I.; Ničeno, B. A Solution to Stefan Problem Using Eulerian Two-Fluid VOF Model. *Brodogr. Teor. Praksa Brodogr. Pomor. Teh*. **2021**, 72, 141–164. https://doi.org/10.21278/brod72408

[31] Mer, S.; Praud, O.; Neau, H.; Merigoux, N.; Magnaudet, J.; Roig, V. The Emptying of a Bottle as a Test Case for Assessing Interfacial Momentum Exchange Models for Euler–Euler Simulations of Multi-Scale Gas-Liquid Flows. *Int. J. Multiph. Flow* **2018**, 106, 109–124. https://doi.org/10.1016/j.ijmultiphaseflow.2018.05.002

[32] Duan, G.; Yamaji, A.; Sakai, M. An incompressible–compressible Lagrangian particle method for bubble flows with a sharp density jump and boiling phase change, *Computer Methods in Applied Mechanics and Engineering*, 372, **2020**, 113425. https://doi.org/10.1016/j.cma.2020.113425

[33] Yang, X.; Kong, S.-C. Smoothed particle hydrodynamics method for evaporating multiphase flow, *Phys. Rev. E* 96 3, 033309, 10, **2017**. https://doi.org/10.1103/PhysRevE.96.033309

[34] Abolmaali, A.M.; Bayat, M.; Hattel, J. Thin-film evaporation in microchannels: A combined analytical and computational fluid dynamics approach to assessing meniscus curvature impact, *International Journal of Thermal Sciences*, 208, **2025**, 109455. https://doi.org/10.1016/j.ijthermalsci.2024.109455

[35] Bureš, L.; Sato, Y.; Direct numerical simulation of evaporation and condensation with the geometric VOF method and a sharp-interface phase-change model, *International Journal of Heat and Mass Transfer*, 173, **2021**, 121233. https://doi.org/10.1016/j.ijheatmasstransfer.2021.121233

[36] Jouhara, H.; Robinson, A. Advances in boiling and evaporative heat transfer science and applications, *International Journal of Heat and Mass Transfer*, 246, **2025**, 127047. https://doi.org/10.1016/j.ijheatmasstransfer.2025.127047

[37] Mialhe, G.; Tanguy, S.; Tranier, L.; Popescu, E.-R.; Legendre, D. An extended model for the direct numerical simulation of droplet evaporation. Influence of the Marangoni convection on Leidenfrost droplet, *Journal of Computational Physics*, 491, **2023**, 112366. https://doi.org/10.1016/j.jcp.2023.112366

[38] Long, T.; Pan, J.; Zaleski, S. An Edge-based Interface Tracking (EBIT) method for multiphase flows with phase change, *Journal of Computational Physics*, 513, **2024**, 113159. https://doi.org/10.1016/j.jcp.2024.113159

[39] Cipriano, E.; Saufi, A.E.; Frassoldati, A.; Faravelli, T.; Popinet, S.; Cuoci, A. Multicomponent droplet evaporation in a geometric volume-of-fluid framework, *Journal of Computational Physics*, 507, **2024**, 112955. https://doi.org/10.1016/j.jcp.2024.112955

[40] Rieckmann, M.; Smuda, M.; Stephan, P.; Kummer, F. The extended Discontinuous Galerkin method for two-phase flows with evaporation, *Journal of Computational Physics*, 499, **2024**, 112716. https://doi.org/10.1016/j.jcp.2023.112716

[41] Szewc, K. Development of Smoothed Particle Hydrodynamics approach for modelling of multiphase flows with interfaces. *PhD Thesis* Université de Lorraine, 2013. https://hal.univ-lorraine.fr/tel-01750701

[42] Amrofel, N.; Dymitrowska, M.; Obliger, A.; Tinet, A.-J.; Golfier, F. Drying in nanoporous media with Kelvin effect: Capillary imbibition against evaporation by smoothed particle hydrodynamics method. *Physics of Fluids* **2024**; 36 (2): 022028. https://doi.org/10.1063/5.0186916

[43] Han, P.; Cong, H.; Zhou, Z.; Sun, Z.; Xi, G. An incompressible–compressible multiphase mesh-free particle method for boiling and condensation simulation. *Physics of Fluids* 1 June **2024**; 36 (6): 063346. https://doi.org/10.1063/5.0212770

[44] Brackbill, J.U.; Kothe, D.B.; Zemach, C. A Continuum Method for Modeling Surface Tension. *J. Comput. Phys.* **1992**, 100, 335–354. https://doi.org/10.1016/0021-9991(92)90240-Y

[45] Torriano, F.; Chaaban, M.; Picher, P. Numerical study of parameters affecting the temperature distribution in a disc-type transformer winding, *Applied Thermal Engineering*, 30, 14–15, **2010**, 2034-2044. https://doi.org/10.1016/j.applthermaleng.2010.05.004

[46] Szewc, K. Smoothed particle hydrodynamics modeling of granular column collapse. *Granular Matter*, **2017**, 19, 3. https://doi.org/10.1007/s10035-016-0684-3

[47] Jiaxin, L.; Tian, Y.; Wang, Z.; Li, L.; Ma, S. Modeling the impact of terrain surface deformation on drag force using discrete element method and empirical formulation, *Applied Mathematical Modelling*, 136, **2024**, 115636. https://doi.org/10.1016/j.apm.2024.115636

[48] Fu-Zhen, C.; Hong-Fu, Q.; Wei-Ran, G. Numerical simulation of heat transfer in gas-particle two-phase flow with smoothed discrete particle hydrodynamics. *Acta Phys. Sin*., **2014**, 63(23): 230206. (in Chinese) https://doi.org/10.7498/aps.63.230206

[49] Fu-Zhen, C.; Hong-Fu, Q.; Wei-Ran, G. Simulation of aerolian sand transport with SPH-FVM coupled method. *Acta Phys. Sin*., **2014**, 63(13): 130202. (in Chinese) https://doi.org/10.7498/aps.63.130202

[50] Werdelmann, B. Ein Verfahren zur Kopplung der Smoothed-Particle-Hydrodynamics- und der Finite-Volumen-Methode. *Dissertation*. Karlsruher Institut für Technologie, **2023**. https://doi.org/10.30819/5689

[51] Ren, B.; Yan, X.; Yang, T. et al. Fast SPH simulation for gaseous fluids. *Vis Comput* 32, 523–534, **2016**. https://doi.org/10.1007/s00371-015-1086-y

[52] Wojtan, C.; Thürey, N.; Gross, M.; Turk, G. Physics-inspired topology changes for thin fluid features. *ACM Trans. Graph*. 29, 4, Article 50 (July **2010**), 8 pages. https://doi.org/10.1145/1778765.1778787

[53] Heiss-Synak, P.; Kalinov, A.; Strugaru, M.; Etemadi, A.; Yang, H.; Wojtan, C. Multi-Material Mesh-Based Surface Tracking with Implicit Topology Changes. *ACM Transactions on Graphics (TOG)*, 43(4), **2024**, 1-14. https://doi.org/10.1145/3658223

[54] Xu, Y. Smoothed Particle Hydrodynamics for Fluid Simulation in Computer Graphics. *PhD Thesis*, **2025**. University of Groningen. https://doi.org/10.33612/diss.1163967456

[55] Ahamed, S. Modeling Drop-Wall Interaction and Thermomechanical Stress in High Temperature Material. *PhD Thesis*, Texas Tech University, 2024. https://hdl.handle.net/2346/100209

[56] Yang, X.; Pan, Y.; Kong, S.-C. Predicting the outcomes of fuel drop impact on heated surfaces using SPH simulation. ICLASS 2018, *14th Triennial International Conference on Liquid Atomization and Spray Systems*, Chicago, IL, USA, July 22-26, **2018**. https://doi.org/10.48550/arXiv.1712.05830

[57] Yang, X.; Kong, S. C. Adaptive resolution for multiphase smoothed particle hydrodynamics. *Computer Physics Communications*, 239, **2019**, 112-125. https://doi.org/10.1016/j.cpc.2019.01.002

[58] Xiong, H.; Zhang, C.; Yu, Z. Multiphase SPH modeling of water boiling on hydrophilic and hydrophobic surfaces, *International Journal of Heat and Mass Transfer*, 130, **2019**, 680-692. https://doi.org/10.1016/j.ijheatmasstransfer.2018.10.119

[59] Zhuang, T.; Wu, J.; Zhang, T.; Dong, X. A weakly compressible smoothed particle hydrodynamics framework for melting multiphase flow. *AIP Advances*, **2022**, 12 (2): 025329. https://doi.org/10.1063/5.0057583

[60] Wickert, D.; Prokop, G. Simulation of water evaporation under natural conditions—A state-of-the-art overview. *Exp. Comput. Multiph. Flow* 3, 242–249, **2021**. https://doi.org/10.1007/s42757-020-0071-5

[61] Pan, Z.; Weibel, J.A.; Garimella, S.V. A saturated-interface-volume phase change model for simulating flow boiling, *International Journal of Heat and Mass Transfer*, 93, **2016**, 945-956. https://doi.org/10.1016/j.ijheatmasstransfer.2015.10.044

[62] Berenson, P. J. "Film-Boiling Heat Transfer From a Horizontal Surface." *ASME. J. Heat Transfer*. August **1961**; 83(3): 351–356. https://doi.org/10.1115/1.3682280

[63] Meier, C.; Fuchs, S.L.; Hart, A.J.; Wall, W.A. A novel smoothed particle hydrodynamics formulation for thermo-capillary phase change problems with focus on metal additive manufacturing melt pool modeling, *Computer Methods in Applied Mechanics and Engineering*, 381, **2021**, 113812. https://doi.org/10.1016/j.cma.2021.113812

[64] Dong, X.; Hao, G.; Yu, R. Two-dimensional smoothed particle hydrodynamics (SPH) simulation of multiphase melting flows and associated interface behavior. *Engineering Applications of Computational Fluid Mechanics*, **2022** 16(1), 588–629. https://doi.org/10.1080/19942060.2022.2026820

[65] Zhang, X.-Y.; Duan, X.-Y.; Zhang, T.; Xu, M.-H.; Sun, S.; Gong, L.; Liu, L. Model and algorithmic augmentation in phase field method for simulating the pool boiling phenomenon with high-density ratio. *Physics of Fluids*, **2025**; 37 (1): 013309. https://doi.org/10.1063/5.0244235

[66] Wang, Z.; Zheng, X.; Chryssostomidis, C.; Karniadakis, G.E. A phase-field method for boiling heat transfer, *Journal of Computational Physics*, 435, **2021**, 110239. https://doi.org/10.1016/j.jcp.2021.110239

[67] Feng, X.; Qiao, Z.; Sun, S.; Wang, X. An energy-stable Smoothed Particle Hydrodynamics discretization of the Navier-Stokes-Cahn-Hilliard model for incompressible two-phase flows, *Journal of Computational Physics*, 479, **2023**, 111997. https://doi.org/10.1016/j.jcp.2023.111997

[68] Ghoneim, A.Y. Meshfree phase-field modeling of three-phase flow using smoothed particle hydrodynamics with differential reproducing kernels and artificial compressibility, *Journal of Computational and Applied Mathematics*, 469, **2025**, 116654. https://doi.org/10.1016/j.cam.2025.116654

[69] Zhou, S., Wang, M.Y. Multimaterial structural topology optimization with a generalized Cahn–Hilliard model of multiphase transition. *Struct Multidisc Optim* 33, 89–111, **2007**. https://doi.org/10.1007/s00158-006-0035-9

[70] Wang, Y.; Hirshikesh; Yu, T.; Natarajan, S.; Bui T.Q. Phase-field method combined with optimality criteria approach for topology optimization, *Applied Mathematical Modelling*, 129, **2024**, 509-521. https://doi.org/10.1016/j.apm.2024.02.006

[71] Perić M. Prediction of cavitation on ships. *Brodogradnja*. **2022**; 73(3):39-58. https://doi.org/10.21278/brod73303

[72] Savio, A.; Cianferra, M.; Armenio, V. Analysis of Performance of Cavitation Models with Analytically Calculated Coefficients. *Energies* **2021**, 14, 6425. https://doi.org/10.3390/en14196425

[73] Romani S.; Morgut M.; Parussini L.; Piller M. Uncertainty quantification and global sensitivity analysis of turbulence model closure coefficients for sheet cavity flow around a hydrofoil. *Brodogradnja*. **2025**; 76(1). https://doi.org/10.21278/brod76105

[74] Aganin, A.A.; Davletshin, A.I. A particle model of interaction between weakly non-spherical bubbles, *Applied Mathematical Modelling*, 126, **2024**, 185-205. https://doi.org/10.1016/j.apm.2023.10.031

[75] Lyu, H.G., Sun, P.N., Colagrossi, A. et al. Towards SPH simulations of cavitating flows with an EoSB cavitation model. *Acta Mech. Sin*. 39, 722158, **2023**. https://doi.org/10.1007/s10409-022-22158-x

[76] Kalateh, F.; Hosseinzadeh, S.; Koosheh, A. Analysis and application of the modified smoothed particle hydrodynamics method to simulate cavitating flow. *J Braz. Soc. Mech. Sci. Eng*. 44, 119, **2022**. https://doi.org/10.1007/s40430-021-03205-z

[77] Joshi, S. Cavitation erosion modelling using SPH. *PhD Thesis*, Materials Science, Université Grenoble Alpes, **2018**. https://theses.hal.science/tel-02014179

[78] Cortez-González, J.; Murrieta-Dueñas, R.; Alvarado-Rodríguez, C.E.; Segovia-Hernández, J.G.; Hernández, S.; Gutiérrez-Guerra, R. New Approach Lagrangian for Numerical Analysis Within L-V One Equilibrium Stage of a Distillation Column Using the SPH Method, *Industrial & Engineering Chemistry Research*, American Chemical Society **2024**, 63, 50, 21974 - 21990. https://doi.org/10.1021/acs.iecr.4c03313

[79] Zadravec, D.; Ferdelji, N.; Lončar, D. *et al*. Towards a comprehensive approach to optimal control of non-ideal binary batch distillation. *Optim Eng* 23, 2111–2141, **2022**. https://doi.org/10.1007/s11081-022-09727-2

[80] Calderon-Sanchez, J.;Cercos-Pita, J.L.; Duque, D. A geometric formulation of the Shepard renormalization factor, *Computers & Fluids*, 183, **2019**, pp. 16-27. https://doi.org/10.1016/j.compfluid.2019.02.020

[81] Fang, X.-L.; Wang, P.-P.; Meng, Z.-F.; Ming, F.-R.; Zhang, A.M. An accurate thermodynamic Riemann-SPH model for multiphase flows with applications in bubble dynamics, *Journal of Computational Physics*, 533, **2025**, 113969. https://doi.org/10.1016/j.jcp.2025.113969

[82] Boniou, V. On the numerical simulation of evaporating two-phase flows using sharp interface capturing methods. *PhD Thesis*, Fluids mechanics. Université Paris-Saclay, **2021**. https://theses.hal.science/tel-03537995v1

[83] Gunstensen, A. K. Lattice-Boltzmann studies of multiphase flow through porous media. *Doctoral dissertation*, Massachusetts Institute of Technology. **1992**. https://dspace.mit.edu/handle/1721.1/13168

[84] Zahedi, S.; Kronbichler, M.; Kreiss, G. Spurious currents in finite element based level set methods for two-phase flow. *Int. J. Numer. Meth. Fluids*, 69, **2012**: 1433-1456. https://doi.org/10.1002/fld.2643

[85] Lafaurie, B.; Nardone, C.; Scardovelli, R.; Zaleski, S.; Zanetti, G. Modelling Merging and Fragmentation in Multiphase Flows with SURFER, *Journal of Computational Physics*, 113, 1, **1994**, 134-147. https://doi.org/10.1006/jcph.1994.1123

[86] Mathieu, B. Études physique, expérimentale et numérique des mécanismes de base intervenant dans les écoulements diphasiques, *thèse de doctorat*, Énergétique Aix-Marseille 1, **2003**. https://theses.fr/2003AIX11039

[87] Toutant, A.; Mathieu, B.; Lebaigue, D. Volume-conserving mesh smoothing for front-tracking methods, *Computers & Fluids*, 67, **2012**, pp. 16-25. https://doi.org/10.1016/j.compfluid.2012.06.019

[88] Chen, R.; Tian, W.; Su, G.H.; Qiu, S.; Ishiwatari, Y.; Oka, Y. Numerical investigation on bubble dynamics during flow boiling using moving particle semi-implicit method, *Nuclear Engineering and Design*, 240, 11, **2010**, 3830-3840. https://doi.org/10.1016/j.nucengdes.2010.08.008

[89] Koshizuka, S.; Oka, Y. Moving particle semi-implicit method: Fully lagrangian analysis of incompressible flows. In *Proceedings of the European Congress on computational methods in applied sciences and engineering (ECCOMAS),* **2000**, September, Barcelona, Spain (pp. 11-14). https://congress2.cimne.com/eccomas/proceedings/eccomas2000/pdf/73.pdf

[90] Gros, E. Numerical Modelling of Two-Phase Flow with Moving Boundary Fitted Meshes. *PhD Thesis*, EPFL, 2018. https://infoscience.epfl.ch/handle/20.500.14299/146720

[91] Messahel, R. ALE and SPH formulations for Fluid Structure Interaction : shock waves impact. *PhD Thesis*, **2016**, Université de Lille. https://pepite-depot.univ-lille.fr/LIBRE/EDSPI/2016/50376-2016-Messahel.pdf

[92] Olivier, G. Anisotropic Metric-Based Mesh Adaptation for Unsteady CFD Simulations Involving Moving Geometries, Adaptation de Maillage Anisotrope Par Prescription de Champ de Métriques Appliquée Aux Simulations Instationnaires En Géométrie Mobile; *PhD Thesis*, Université Pierre et Marie Curie—Paris VI: Paris, France, **2011**. https://theses.hal.science/tel-00739406v1

[93] Barral, N. Time-Accurate Anisotropic Mesh Adaptation for Three-Dimensional Moving Mesh Problems, Adaptation de Maillage Anisotrope Dépendant Du Temps Pour Des Problèmes Tridimensionnels En Maillage Mobile; *PhD Thesis*, Université Pierre et Marie Curie—Paris VI: Paris, France, **2015**. https://theses.hal.science/tel-01284113/

[94] Ducobu, F.; Arrazola, P.-J.; Rivière-Lorphèvre, E.; Ortiz de Zarate, G.; Madariaga, A.; Filippi, E. The CEL Method as an Alternative to the Current Modelling Approaches for Ti6Al4V Orthogonal Cutting Simulation, *Procedia CIRP*, 58, **2017**, pp. 245-250. https://doi.org/10.1016/j.procir.2017.03.188

[95] Noh, W. F. CEL: A time-dependent, two-space-dimensional, coupled Eulerian-Lagrange code (No. UCRL-7463). Lawrence Livermore National Lab.(LLNL), Livermore, CA (United States). **1963**. https://www.osti.gov/servlets/purl/4621975

[96] Sha, S.; Dyson, A.P.; Kefayati, G.; Tolooiyan, A. Modelling of debris flow-boulder-barrier interactions using the Coupled Eulerian Lagrangian method, *Applied Mathematical Modelling*, 127, **2024**, pp. 143-171. https://doi.org/10.1016/j.apm.2023.11.029

[97] Shi, H.; Huang, Y.; Feng, D. Numerical investigation on the role of check dams with bottom outlets in debris flow mobility by 2D SPH. *Sci Rep* 12, 20456, **2022**. https://doi.org/10.1038/s41598-022-24962-4

[98] Ng, C.W.W.; Li, Z.; Wang, Y.D.; Liu, H.; Poudyal, S.; De Silva, W.A.R.K. Influence of deflector on the impact dynamics of debris flow against rigid barrier, *Engineering Geology*, 321, **2023**, 107135. https://doi.org/10.1016/j.enggeo.2023.107135

[99] Tensi, H.M. Wetting Kinematics. In: Liščić, B., Tensi, H.M., Luty, W. (eds) Theory and Technology of Quenching. Springer, Berlin, Heidelberg. **1992**. https://doi.org/10.1007/978-3-662-01596-4_5

[100] Felde, I.; Shi, W. Hybrid Approach for Solution of Inverse Heat Conduction Problems. In: *Proceedings of the 2014 IEEE International Conference on Systems*, Man, and Cybernetics (SMC), San Diego, CA, USA, 5–8 October **2014**; IEEE: Piscataway, NJ, USA, 2014; pp. 3896–3899. https://ieeexplore.ieee.org/document/6974539

[101] Breinlinger, T.; Polfer, P.; Hashibon, A.; Kraft, T. Surface tension and wetting effects with smoothed particle hydrodynamics, *Journal of Computational Physics*, 243, **2013**, pp 14-27. https://doi.org/10.1016/j.jcp.2013.02.038

[102] Olejnik, M.; Pozorski, J. A Robust Method for Wetting Phenomena Within Smoothed Particle Hydrodynamics. *Flow Turbulence Combust* 104, 115–137, **2020**. https://doi.org/10.1007/s10494-019-00048-6

[103] Eyal, O.; Brauner, N.; Ullmann, A.; Goldstein, A. Slip boundary condition near the wall-interface contact line in axial stratified two-phase flow, *Applied Mathematical Modelling*, 126, **2024**, pp. 828-853. https://doi.org/10.1016/j.apm.2023.11.008

[104] Nichita, B.A. An Improved CFD Tool to Simulate Adiabatic and Diabatic Two-Phase Flows, *PhD Thesis*, EPFL: Lausanne, Switzerland, **2010**. 10.5075/epfl-thesis-4776

[105] Liu, X. Development of Particle and Mesh-Particle Hybrid Methods for Multi-Phase Flow Simulations. *PhD Thesis*, **2015**, Kyushu University. https://hdl.handle.net/2324/1543949

[106] Zhu, X. An energy stable SPH method for fluid flow in geological porous media, *PhD Thesis*, KAUST Research Repository, **2024**. https://doi.org/10.25781/KAUST-Q4365

[107] Zhu, X.; Wang, X.; Kou, J.; Sun, S. An energy stable incompressible SPH method with consistent solid boundary treatment, *Journal of Computational and Applied Mathematics*, 436, **2024**, 115367. https://doi.org/10.1016/j.cam.2023.115367

[108] Lyu, S.; Wang, K.; Zhang, Z.; Pedrono, A.; Sun, C.; Legendre, D. A hybrid VOF-IBM method for the simulation of freezing liquid films and freezing drops, Journal of Computational Physics, 432, 2021, 110160. https://doi.org/10.1016/j.jcp.2021.110160

[109] Monaghan, J. J.; Kocharyan, A. SPH simulation of multi-phase flow. *Computer Physics Communications*, 87(1-2), **1995** 225-235. https://doi.org/10.1016/0010-4655(94)00174-Z

Submitted: 20.5.2025

Accepted: 11.7.2025

Alen Cukrov*
Ivanka Boras
ivanka.boras@fsb.unizg.hr
University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Zagreb, Croatia

Josip Bašić
University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, Split, Croatia
jobasic@fesb.hr

*Corresponding author:
alen.cukrov@fsb.unizg.hr