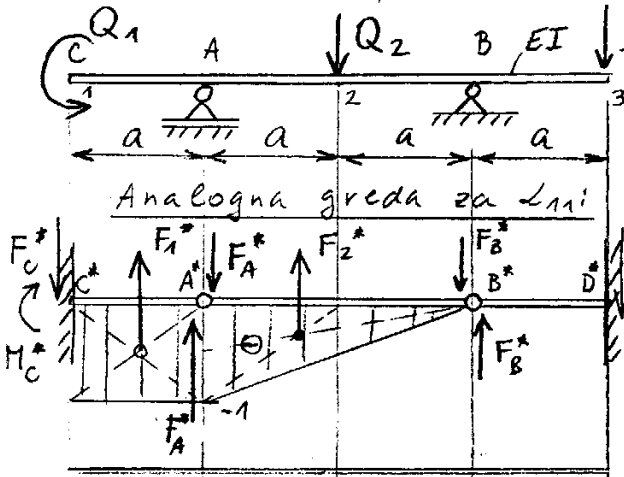
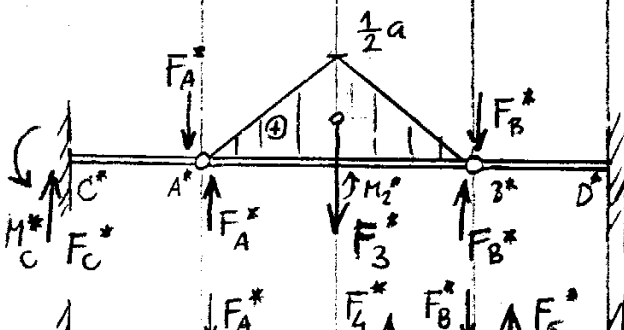
Za nosač zadan i opterećen prema slici izračunati uplivne koeficijente: α_{11}, α_{12}, α_{13}, α_{22} i α_{33}.

Zadano: a, EI.

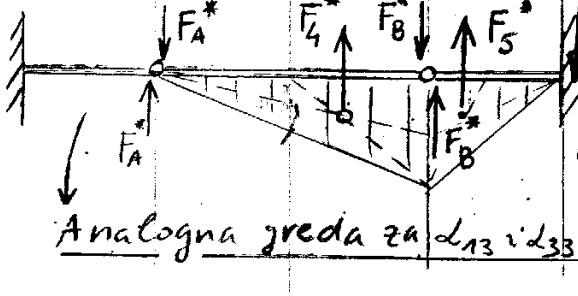


Analogna greda za α_{11}:

Analogna greda za α_{12} i α_{22}:



Analogna greda za α_{13} i α_{33}:



α_{11}: $(Q_1 = 1,\ Q_2 = Q_3 = \phi)$

$\Sigma M_A = \phi \quad F_B \cdot 2a - 1 = \phi \Rightarrow$

$F_B = \frac{1}{2a}$; $F_A = \frac{1}{2a}$

$F_1^* = 1 \cdot a = a$; $F_2^* = \frac{1}{2} \cdot 1 \cdot 2a = a$

$\Sigma M_B^* = \phi \quad F_2^* \cdot \frac{2}{3} 2a - F_A^* \cdot 2a = \phi \ / : 2a$

$F_A^* = \frac{2}{3} F_2^* = \frac{2}{3} a$

$F_C^* = F_1^* + F_A^* = a + \frac{2}{3} a = \frac{5}{3} a$

$Q_C^* + F_C^* = \phi \Rightarrow Q_C^* = -F_C^* = -\frac{5}{3} a$

$\alpha_{11} = -\frac{Q_C^*}{EI} = \boxed{\frac{5}{3} \frac{a}{EI}} \left[\frac{rad}{Nm}\right] \rightarrow \left[(Nm)^{-1}\right]$

α_{12} i α_{22}: $(Q_2 = 1,\ Q_1 = Q_3 = \phi,$

$\Sigma M_A = \phi \quad F_B \cdot 2a - 1 \cdot a = \phi$

$F_B = \frac{1}{2}$; $F_A = \frac{1}{2}$; $F_3^* = \frac{1}{2} 2a \cdot \frac{1}{2} a = \frac{1}{2}$

$\Sigma M_B^* = \phi \quad F_A^* \cdot 2a - F_3^* \cdot a = 0 \ / : a$

$F_A^* = F_B^* = \frac{1}{2} F_3^* = \frac{1}{2} ; \frac{1}{2} a^2 = \frac{1}{4} a^2$

$F_C^* = F_A^* = \frac{1}{4} a^2$

$M_C^* = F_A^* \cdot a = \frac{1}{4} a^3$

$M_2^* = F_A^* a - \frac{1}{2} a \cdot \frac{1}{2} a \cdot \frac{1}{3} a = \frac{1}{4} a^3 - \frac{1}{12} a^3 = \frac{3-1}{12} a^3 = \frac{1}{6} a^3$

$\alpha_{12} = -\frac{Q_C^*}{EI} = \boxed{-\frac{1}{4} \frac{a^2}{EI}} [N^{-1}]$; $\alpha_{22} = \frac{M_2^*}{EI} = \boxed{\frac{1}{6} \frac{a^3}{EI}} \left[\frac{m}{N}\right]$

α_{13} i α_{33}: $(Q_3 = 1;\ Q_1 = Q_2 = \phi)$ $\Sigma M_B = \phi \quad F_A \cdot 2a - 1 \cdot a = \phi \Rightarrow$

$F_A = \frac{1}{2}$; $F_B = \frac{3}{2}$

$F_4^* = \frac{1}{2} 2a \cdot 1 \cdot a = a^2$; $F_5^* = \frac{1}{2} a \cdot 1 \cdot a = \frac{1}{2} a^2$

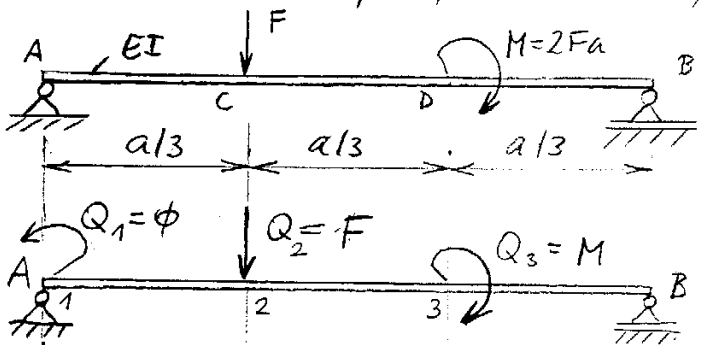
$\Sigma M_B^* = \phi \Rightarrow F_A^* = \frac{1}{3} F_4^* = \frac{1}{3} a^2 = F_C^*$; $M_C^* = F_A^* \cdot a = \frac{1}{3} a^3$

$\Sigma F_z^* = \phi \Rightarrow F_B^* = \frac{2}{3} a^2$; $M_D^* = a^3$; $Q_C^* = -F_C^*$ $\alpha_{13} = -\frac{Q_C^*}{EI} = \boxed{\frac{1}{3} \frac{a^2}{EI}} [N^{-1}]$

$\alpha_{33} = \frac{M_D^*}{EI} = \boxed{\frac{a^3}{EI}} \left[\frac{m}{N}\right]$

Metodom upljivih koeficijenata naći w_C i α_A.

Zadano: F; a; $M = 2Fa$; EI



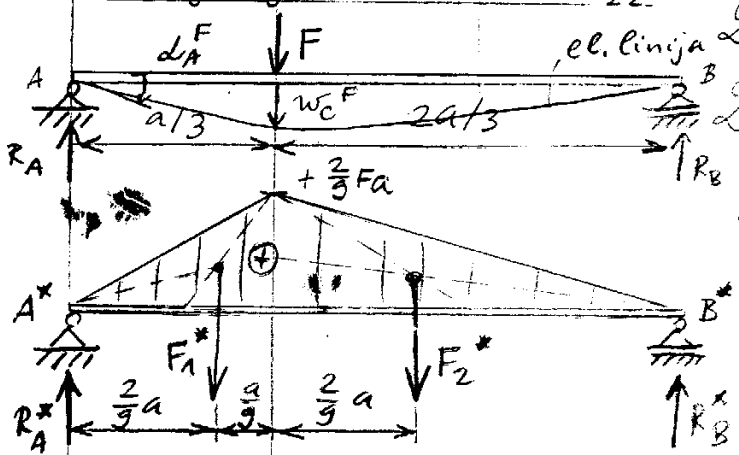
$Q_1 = \phi$; $Q_2 = F$; $Q_3 = M = 2F$

$$\mathcal{Q}_1 = \alpha_{11}\overset{\phi}{Q_1} + \alpha_{12}Q_2 + \alpha_{13}Q_3$$

$$\mathcal{Q}_2 = \alpha_{21}\overset{\phi}{Q_1} + \alpha_{22}Q_2 + \alpha_{23}Q_3$$

$$\mathcal{Q}_3 = \alpha_{31}\overset{\phi}{Q_1} + \alpha_{32}Q_2 + \alpha_{33}Q_3$$

Analogna greda za α_{12} i α_{22}:

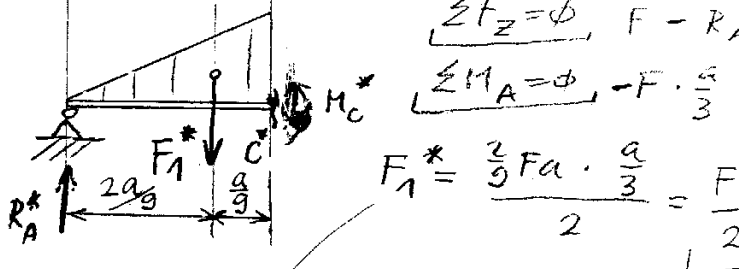


$$\mathcal{Q}_1 = \alpha_{12}Q_2 + \alpha_{13}Q_3$$

$$\mathcal{Q}_2 = \alpha_{22}Q_2 + \alpha_{23}Q_3$$

$$\mathcal{Q}_1 = \alpha_A \; ; \; \mathcal{Q}_2 = w_C$$

$\boxed{\alpha_{12},\ \alpha_{13},\ \alpha_{22},\ \alpha_{23} = ?}$



$\Sigma F_z = \phi$, $F - R_A - R_B = \phi$ (1) $\Rightarrow R_A = \frac{2}{3}F$

$\Sigma M_A = \phi$, $-F \cdot \frac{a}{3} + R_B \cdot a = \phi$ (2) $\Rightarrow R_B = \frac{F}{3}$

$$F_1^* = \frac{\frac{2}{9}Fa \cdot \frac{a}{3}}{2} = \frac{Fa^2}{27}; \quad F_2^* = \frac{\frac{2}{9}Fa \cdot \frac{2a}{3}}{2} = \frac{2Fa^2}{27}$$

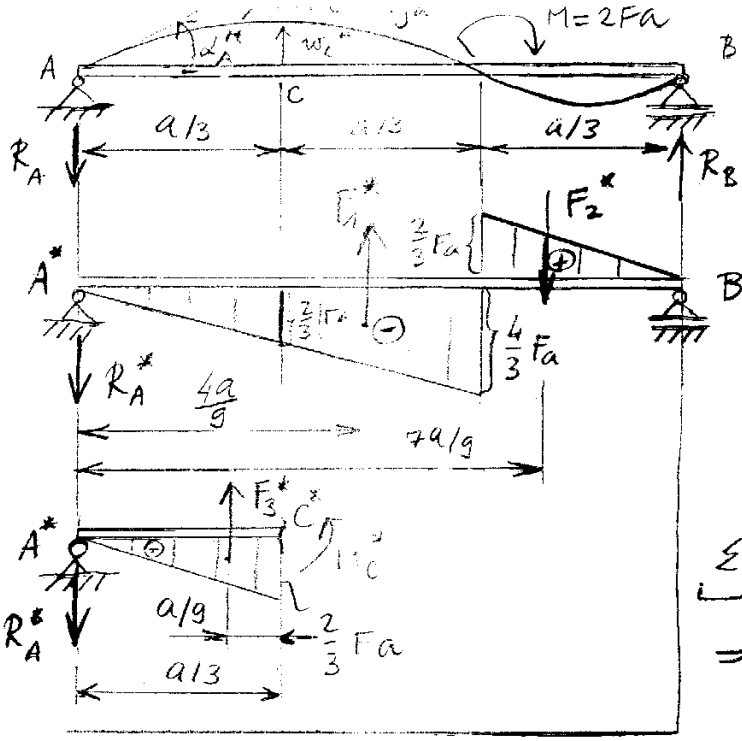
$\Sigma F_z^* = \phi$, $F_1^* + F_2^* - R_A^* - R_B^* = \phi$ (1) $\Rightarrow R_A^* = \frac{5}{81}Fa^2$

$\Sigma M_A^* = \phi$, $F_1^* \cdot \frac{2}{9}a + F_2^* \cdot \frac{5}{9}a - R_B^* \cdot a = \phi$ (2) $\Rightarrow R_B^* = \frac{4}{81}Fa^2$

$$\alpha_A^F = -\frac{Q_A^*}{EI} = -\frac{R_A^*}{EI} = -\frac{5}{81}\frac{Fa^2}{EI} \Rightarrow \boxed{\alpha_{12} = \frac{\alpha_A^F}{F} = -\frac{5}{81}\frac{a^2}{EI}}$$

$$w_C^F = \frac{M_C^*}{EI} = \frac{R_A^* \cdot \frac{a}{3} - F_1^* \frac{a}{9}}{EI} = \frac{\frac{5}{81}Fa^2 \cdot \frac{a}{3} - \frac{Fa^2}{27} \cdot \frac{a}{9}}{EI} \Rightarrow$$

$$w_C^F = \frac{4}{243}\frac{Fa^3}{EI} \Rightarrow \boxed{\alpha_{22} = \frac{w_C^F}{F} = \frac{4}{243}\frac{a^3}{EI}}$$



$\Sigma F_z = 0 \quad R_A = R_B$

$\Sigma M_A = 0 \quad R_B \cdot a = M = 2Fa$

$\Rightarrow R_B = R_A = \underline{2F}$

$F_1^* = \frac{\frac{4}{3}Fa \cdot \frac{2}{3}a}{2} = \underline{\frac{4}{9}Fa^2}$

$F_2^* = \frac{\frac{2}{3}Fa \cdot \frac{a}{3}}{2} = \underline{\frac{1}{9}Fa^2}$

$\Sigma M_A^* = 0 \quad F_1^* \cdot \frac{4}{9}a - F_2^* \cdot \frac{7}{9}a - R_B^* \cdot a =$

$\Rightarrow \underline{R_B^* = \frac{1}{9}Fa^2}$

$\Sigma F_z^* = 0 \quad F_2^* - F_1^* + R_A^* + R_B^* = 0 \Rightarrow \underline{R_A^* = \frac{2}{9}Fa^2}$

$\alpha_A^M = -\frac{Q_A^*}{EI} = -\frac{-R_A^*}{EI} = \frac{2}{9}\frac{Fa^2}{EI} \Rightarrow \alpha_{13} = \frac{\alpha_A^M}{M=2Fa} = \boxed{\frac{1}{9}\frac{a}{EI}}$

$F_3^* = \frac{\frac{2}{3}Fa \cdot \frac{a}{3}}{2} = \frac{Fa^2}{9}$; $M_C^* = F_3^* \cdot \frac{a}{9} - R_A^* \cdot \frac{a}{3} = \underline{-\frac{5}{81}Fa^3}$

$w_C^M = \frac{M_C^*}{EI} = \frac{5}{81}\frac{Fa^3}{EI} \Rightarrow \alpha_{23} = \frac{w_C^M}{M=2Fa} = \boxed{-\frac{5}{162}\frac{a^2}{EI}}$

$\alpha_A = \alpha_{12} \cdot F + \alpha_{13} M = -\frac{5a^2}{81} \cdot F + \frac{1}{9}\frac{a}{EI} \cdot 2Fa = \frac{13}{81}\frac{Fa^2}{EI}$

$$\boxed{\alpha_A = \frac{13}{81}\frac{Fa^2}{EI}}$$

$w_C = \alpha_{22} \cdot F + \alpha_{33} \cdot M = \frac{4a^3}{243EI} \cdot F - \frac{5}{162}\frac{a^2}{EI} \cdot 2Fa$

$$\boxed{w_C = -\frac{11}{243} \cdot \frac{Fa^3}{EI}}$$