Asymptotic analysis and enriched finite elements for the modeling of defects in three-dimensional mechanical structures. Application to lifetime assessment of structures

---

Involved laboratories : Laboratoire Roberval, UMR 7337, Université de Technologie de Compiègne
Contacts: Eduard Marenić, UTC (eduard.marenic@utc.fr)
Delphine Brancherie, UTC (delphine.brancherie@utc.fr)

Position: master position funded for 6 months (or less)
Keywords: Asymptotic analysis, rupture, finite elements, boundary elements
Required skills: Mechanics of structure. Applied mathematics. Scientific computing.

---

1 Context

When studying the behavior of mechanical structures, the global response - sufficient for current applications is usually obtained considering that the material is perfect, ignoring the possible presence of defects. Nevertheless, some applications require finer analyses, for instance when considering the behavior till rupture of structures. The objective here is to design a numerical tool capable of predicting the severity of a defect in terms of its size and position in the structure with low computational cost that is without a fine description of the geometry. The influence of small defects (heterogeneities or porosities) on the response of mechanical 3D structures is obtained by appealing to numerical strategies based on a coarse description of the geometry and involving both finite element enrichment and asymptotic analysis.

2 Description of the project and objectives

The role played by defects in the initiation and development of rupture is crucial and has to be taken into account in order to realistically describe the behavior till complete failure. Indeed, the presence of porosities or inclusions induces local modifications of the stress field, and this explains that, crack initiation sites are experimentally known to be usually located at the vicinity of those heterogeneities.

The difficulties in that context lie, on one hand, on the fact that those defects are related to scales much smaller than the scale of the structure, and on the other hand, on the random nature of their position and size. Even in a purely deterministic approach, taking those defects into consideration by standard models imposes to resort to geometrical discretisations at the scale of the defects leading to very costly computations.

The objective here is to design a numerical tool capable of predicting the severity of a defect in terms of its size and position in the structure with a low computational cost. More precisely, the proposed strategy aims at taking into account the modification induced by the presence of the small defects through an enrichment of the finite element functions obtained from an asymptotic analysis. Such an approach would allow to assess the criticality of defects by introducing virtual micro-defects with varying positions, sizes and mechanical properties.

The proposed work aims at developing a strategy to take into account material defects (heterogeneities or porosities) by appealing to numerical strategies based on a coarse description of the geometry. This is achieved by modifying the nominal solution obtained on the sound structure by profiles computed from the asymptotic analysis.

Program

This work is based on two main challenges on which is built the internship program:

- development of asymptotic approximations. This part will be performed on the basis of the developments implemented during a previous work. Formulation, for different types of defects, of the approximated problem solved by the correctors. Implementation and comparison of different numerical strategies to solve the previous problem (Finite Element methods with adapted boundary conditions (FEM), integral equations (BEM), coupling between FEM and BEM).
- application to mechanical structures. The developed methodology will be carried out on mechanical structures. One of the envisaged application is the computation of casting products. Due to the process, castings present different types of defects (porosities, heterogeneities, ...). Those defects could be critical as regards the rupture of the considered structure. The proposed strategy, based on a nominal model of the structure, should allow to assess the criticality of defects detected by control tools such as tomography but it should also provide a "map of criticality" for non detectable defects (due to their very small size).

Academic context

This project is part of the ARAMIS project funded by ANR (French National Research Agency).

Bibliography

[1] Bonnaillie-Noël, V., Brancherie, D., Dambrine, M., Tordeux, S., Vial, G. Effect of micro-defects on structure failure: coupling asymptotic analysis and strong discontinuity. *Eur. J. Comput. Mech.*, **19**, 165–175 (2010).

[2] Bonnaillie-Noël, V., Brancherie, D., Dambrine, M., Vial, G. Artificial boundary conditions to compute correctors in linear elasticity. *Numer. Anal. Appl.*, **5**, 129–135 (2012).

[3] Bonnet, M. *Boundary integral equation methods for solids and fluids*. John Wiley & Sons (1999).

[4] Brancherie, D., Dambrine, M., Vial, G., Villon, P. Effect of surface defects on structure failure : a two-scale approach. *Eur. J. Comput. Mech.*, **17**, 613–624 (2008).

[5] Bonnet, M. Fast identification of cracks using higher-order topological sensitivity for 2-D potential problems. *Eng. Anal. Bound. Elem.*, **35**, 223–235 (2011).