Leonhard Euler (Basel, 1707. - St. Petersburg, 1783.) was Swiss mathematician and physicist. Considered to be one of the greatest mathematicians of all time, he produced outstanding mathematics at an outrageous rate with more publishing than any other mathematician before or after him [1]. His creativity was so outstanding that he didn't only deal with mathematics but also with physics and astronomy, solving all kind of problems and bringing new ideas. Considering his contribution to classical mechanics, it is remarkable that he practically by himself established the basic equations for rigid, fluid and deformable bodies. With the thinking ahead of his time, Leonhard Euler was an artist, an explorer, an inventor, a creative person. With an enthusiasm that resonates two centuries and more, Euler's contribution to science can be regarded as fundamental and profound nowadays.

Key words: Euler; mathematics; physics; equations

1. Introduction

In this essay Euler's life will be presented, to show his extraordinary productivity and creativity in which he also set the foundations for some science fields that will develop after his life. He produced classic texts in algebra, differential and integral calculus, calculus of variations etc. First, biographical sketch will be given so that one can capture the life of this creative man, how the environment, especially J. Bernoulli encouraged and tutored young Euler through his education. Then some of his greatest contributions will be presented, as the one relevant in classical mechanics, to show his creativity and how a creative individual can overcome health issues and unpleasant circumstances to keep his spirit undisturbed in the world of ideas.

2. Biography

Euler's life as scientist can be divided into three periods, life in St. Petersburg, in Berlin and again in St. Petersburg. That is how his life will be sketched in this chapter, from early life then life through living in these three cities as three periods.

2.1 Early life

Leonhard Euler was born on April 15, 1707 near Basel, Switzerland as the first child of Paulus Euler and Margarethe Brucker. His father Paulus was a Protestant clergyman, at the time a vicar at the church of St. Jakob. He went to school in Riehen that was not especially good, but his father gave him private lessons in mathematics as his father studied mathematics for two years, taking lectures from Jakob Bernoulli. In his early life, L. Euler showed an extraordinary memory, gift for languages and was able to perform some complicated arithmetic
operations in his head. In the sense of creativity, as a child Euler already showed the potential, high intelligence, to be creative someday. At the age of thirteen Leonhard enrolled at the University of Basel at the philosophical faculty. There he took the courses on elementary mathematics given by Johann Bernoulli, the younger brother of already mentioned Jakob Bernoulli. With his great understanding of mathematics, young Euler soon caught the attention of Bernoulli, who soon became Euler's guide. Bernoulli suggested him mathematical readings and made himself available for discussion with Euler every Saturday afternoon. Euler himself later said that that was the best method for succeeding in mathematical subjects. Soon Bernoulli started admiring young Euler. Later on, Euler entered the theological faculty by the wish of his parents, however Leonhard devoted most of his time to mathematics and it wasn't long when they all agreed that Leonhard carrier should be as a mathematician. Leonhard's progress was rapid. At the age of nineteen he competed with the great scientists by responding to a prize question of the Paris Academy of Sciences about placement of the masts on sailing ship. He was a respectable second [2]. That showed how much confidence he had, although at first nobody believed he could achieve something in that competition, but as a creative one, he relied on his intuition. A year later he applied for the position of physics chair at the University of Basel, with the full support of J. Bernoulli, but he didn't get it because of his youth and lack of publications. A year later Leonhard got the invitation on St. Petersburg’s Academy, mainly due to Johann's son Daniel Bernoulli who had the position there. Even though he was invited for the physiology/medicine position, and Euler accepted, upon his arrival in 1727 he was reassigned to physics rather.

2.2 St. Petersburg Academy, 1727. - 1741.

Euler adjusted easily and quickly to the new life in northern part of Europe. He quickly mastered the Russian language, both in writing and speaking. At first, he resided at the home of Daniel Bernoulli but in 1733 Bernoulli left for an academic post in Switzerland. Then in 1734 Leonhard married Katharina Gsell, the daughter of a Swiss painter teaching at the Academy and they moved into a house of their own. They, the Eulers had thirteen children but as that was a common case then, only five of them survived to adolescence and three of them outlived the parents. Euler’s intellectual life flourished at the Academy, he devoted him to research and it was a period of extraordinary productivity and creativity. For example, he brought a solution of the so called "Basel problem" which is one of his greatest achievements. Considering productivity, he published papers in the journal of the Academy where half of the publications were his. In 1738 he suffered from kind of infection due to which he lost his sight on the right eye [2]. Still that did not affect his productivity either creativity. He kept researching and it was during this period when he wrote Mechanica, also called “a landmark in the history of physics.” Also, in the meantime he worked as a scientific consultant to the government, where he prepared maps, advised the Russian navy and so on. One can admire how Euler handled all variety of scientific and practical problems. Due to the death of the czarina Anna Ivanovna and political turmoil in Russia, Euler decided to leave St. Petersburg in 1741. He took his family to Berlin where he got proposal from the Prussian king Frederick the Second to come and help establish an Academy of Sciences there.

2.3 Berlin 1741. - 1766.

Euler came to Berlin accepting an offer from the Prussian king to help establish an Academy of Sciences. Even though he got many obligations on the Academy such as dealing with personnel matter, oversee the Academy's observatory etc., his mathematical productivity did not slow down. During his stay in Berlin he published two of his greatest works. In 1748, the "Introductio in analysis in infinitorum" where he developed the concept of function as we
know it [1]. The second work was in 1755, a volume on differential calculus, the "Institutiones calculi differentialis". In these two works it can be seen how deep Euler's thinking was, complex and how he relied in his intuition that brought forward something new. Also, while in Berlin, Euler was asked to instruct the Princess of Anhalt Dessau in elementary science. That resulted with the compilation of over 200 letters that were combined in a multivolume masterpiece named "Letters of Euler on Different Subjects in Natural Philosophy Addressed to a German Princess". That letters eventually became an international hit, translated into many languages. Although Euler did great in Berlin, he had a difficult relationship with Frederick the Second. Later decisions were also maybe affected with the fact that Euler has never been offered the presidency of the Berlin Academy even though Jean le Rond d'Alembert recommended Euler while he was asked for an opinion from the king. This, together with other royal rebuffs finally led Euler to leave Berlin in 1766 for St. Petersburg, where he got the invitation to return to the Academy in St. Petersburg. Because of new Empress Catherine the Second also the Great, Euler was more than happy and accepted the invitation.

2.3 St. Petersburg 1766. - 1783.

In St. Petersburg the Academy welcomed back the greatest mathematician in the world. This time he will stay there for good. Two personal tragedies happened there. One was in 1773 when his wife Katharina died. Euler remarried three years later. Second personal tragedy was that in 1771 when he suffered the illness on his second eye (left) also, and he remained almost blind. He could read maybe something in very big font, but he had to have someone to read to him and write for him. Still that did not discourage him and his productivity even gained a boost. Maybe that is how someone truly creative reacts to extreme difficulties! He was producing a paper per week. Also, he wrote an influential textbook on algebra, a 775 page treatise on the motion of the moon and a massive three volume development of integral calculus, the "Institutiones calculi integralis". He was the great example of the triumph of human spirit. On a September day of 1783. he suffered from severe hemorrhage due to which he died immediately. He was laid to rest in St. Petersburg. Euler left behind so many unpublished papers that the Academy was still publishing the backlogs of his papers 48 years after his death.

3. Contribution to science

Here some basic achievements and discoveries by Euler will be presented, both in mathematics and classical mechanics. Since his work is so vast, these two fields are chosen as relevant to the author. Through this chapter one can get a glimpse of how creative Euler was, with his deep thinking about problems and solutions ahead of his time, we can say.

3.1 Contribution to mathematics

As already mentioned above, Euler dealt with all kinds of mathematics: algebra, differential and integral calculus, calculus of variations etc.

Regarding Number Theory, Euler "finished" the work of Euclid considering perfect numbers, so there is Euler-Euclid Theorem where Euclid in his ninth book gave proof that $2^n - 1$ is perfect number when $2^n - 1$ is a prime but Euler proved that these are only the even perfect numbers. So the theorem is [1]: "An even positive integer is a perfect number, that is, equals the sum of its proper divisors, if and only if it has the form $2^{n-1}(2^n-1)$, for some $n$ such that $2^n-1$ is a prime."
Euler solved the Basel problem also, the problem that had stumped the leading mathematicians of the time like Leibniz, Stirling, all the Bernoullis etc. The problem was of determining the sum of the reciprocal squares [2].

\[ 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots \]  

(3.1)

In 1735 Euler found the answer to this problem and it was a genuine surprise showing off his brilliancy and creativity, for the series the sum is \( \pi^2/6 \) [3]. Euler was really fascinated with the theory of numbers and prime numbers, where he also discovered a product formula connecting prime numbers with the zeta function which is by some considered the beginning of "analytic number theory". Then there is also the discovery of gamma function which arisen due to a problem of interpolating the sequence of factorials. The gamma function became one of the most fundamental functions in analysis also in which appears something often referred to as the second Eulerian integral, while the first integral is also called the beta function which in 20th century became relevant in physics of elementary particles.

Euler's constant \( \gamma \) is by some undoubtedly the third most important constant. Euler's constant arises in connection with the harmonic series \( \zeta(1) \) and is defined as the limit [2]

\[ \gamma = \lim_{n \to \infty} \left( 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} - \ln n \right) = 0.57721\ldots \]

(3.2)

Euler's constant is the most mysterious one. It is not even known either it is rational, even though most likely it is not. While defining his constant and showing that it is finite, Euler put in evidence of the divergence of the harmonic series, but also its logarithmic rate of divergence.

In 1748 Euler published the "Introductio in analysin infinitorum" which became one of the most influential mathematical books of all time. Historian Carl Boyer wrote that it was this work (the book) which made the function concept basic in mathematics. Before Euler analysis was about properties of curves, after was about properties of functions. There he defined the polynomials, the trigonometric functions and the exponential functions. Although log tables were devised a century before Euler, he defined the logarithmic function, perceived the significance of the natural logarithm, and applied his results to theoretical matters.

Euler also dealt with complex numbers. His paper from 1749 titled "Recherches sur les racines imaginaires des equations" serves even to the present day as an excellent introduction to the complex variables, in which he made a prescription for finding roots for complex numbers. There he deducted one of the most extraordinary identity in all of mathematics [4]

\[ e^{i\pi} + 1 = 0 \]

(3.3)

This equation is called "the most profound mathematical statement ever written", "filled with cosmic beauty", someone also asked "What could be more mystical than an imaginary number interacting with real numbers to produce nothing?" [4]. Truly the beauty of a creative mind. The equation contains nine basic concepts of mathematics in a single expression: e (the base of natural algorithm), the exponent operation, \( \pi \), plus, multiplication, imaginary unit, equality, one and zero. But before that Euler succeeded in defining the exponential function for complex number and discovered its relationship with the trigonometric functions, as follows [1]

\[ e^{i\theta} = \cos(\theta) + i \cdot \sin(\theta) \]

(3.4)

Also he defined \( i \) as imaginary unit.

His second discovery that comes almost equally mathematically beautiful is Polyhedral Formula. The formula applies to convex polyhedrons which consist of straight edges and flat faces. The formula is [2]
\[ V - E + F = 2 \]  
(3.5)

Where \( V \) stands for vertices, \( E \) for edges and \( F \) for faces.

The "Integral Calculus" is a huge foray into the realm of quadrature and differential equations. It is a three-volume work. In the second volume he presents, among other things, the significance of linear second order differential equations and in third volume a treatment of linear partial differential equations. Euler's approximate method for solving arbitrary first order differential equations and Taylor series method are embedded in first volume.

These chapter covered only a fraction of Euler's contribution to the mathematics, but there is so much more. Through his masterpieces one can sense his creative nature, which went into unknown and explored. Next chapter will stress out some of his contribution to the classical mechanics.

3.2 Contribution to the classical mechanics

Euler's fundamental contribution to particle dynamics can be found in his “Mechanica sive motus scientia analytice exposita”, published in 1736. It is important to point out that the concept of force in Euler’s mechanics was taken from Galileo. In addition, Euler distinguished absolute forces from the gravity of relative forces, which depend upon the relative velocities among the bodies involved. In a work called “Discovering a new mechanical principle”, published in 1752 he showed the equations [3]:

\[ F_x = M \cdot a_x, F_y = M \cdot a_y, F_z = M \cdot a_z \]  
(3.6)

Where \( M \) is the mass. These equations are based on Newton's second law, so these equations are called Newton-Euler equations.

Euler introduced in the rigid body dynamics the concept of moment of inertia of rigid body, which simplifies the language and the solution of problems. He also calculated the moments of inertia of several homogeneous bodies. In addition, he adopted system of reference attached to the rigid body and discovered the principal axes of inertia.

In 1755 Euler prepared for the Berlin Academy the “Principes généraux de l’état d’équilibre des fluides”, which discussed fluid equilibrium. He considered two types of fluids, the compressible and the incompressible, both submitted to any system of forces. The mass of the fluid is taken into account assuming a 3D infinitesimal parallelepiped with dimensions \( dx, dy, dz \). If the forces components acting on the body are \( P, Q, R \) and the body density is \( q \), then the fluid element of volume \( dx dy dz \) has force components \( Pqdx dy dz, Qqdx dy dz \) and \( Rqdx dy dz \). Euler derived from this a general equilibrium equation [3]:

\[ \frac{dp}{q} = P dy + Q dy + Rdz \]  
(3.7)

The forces \( P, Q, R \) must be such that the differential form \( Pdx + Qdy + Rdz \) either becomes integrable when the density \( q \) is constant or uniquely dependent on the elasticity \( p \), or becomes integrable when multiplied by some function.

Euler considered the original state of the fluid as the configuration of particles and their velocities. This state is assumed to be known at a given time as well as the forces acting on it. The problem to be solved is to calculate at any time the pressure in each point of the fluid, the density and the velocity of the fluid elements that pass through that point. In a paper called “Principia motus fluidorum”, Euler explains the problem of the kinematics of continuous media. He calculates the form, which the elementary parallelepiped, whose origin is \( Z \) and sides \( dx, dy, dz \), will assume at time \( t + dt \) because of the fluid motion. Euler equations express the
conservation of mass, momentum and energy and these constitute a system of nonlinear hyperbolic partial differential equations. Lagrange wrote about Euler's work on fluid: “Through Euler’s discovery the whole of fluid mechanics was reduced to a matter of analysis alone, and if the equations which contain it were integrable, in all cases the circumstances of the motion and behavior of a fluid moved by any forces could be determined. Unfortunately, they are so difficult that, up to the present, it has only been possible to succeed in very special cases.” Euler indicated the simplicity that results if \( u\,dx + v\,dy + w\,dz \) is a complete differential. Much later this was distinguished as the case in which a velocity potential existed, or the case of irrotational motion.

Euler also applied the calculus of variations to elasticity theory, specifically to the bending of a rod subjected to an axial load. He came up with the equation that describes the relationship between the beam's deflection and the applied load [2].

Euler developed optimal profiles for teeth in cogwheel that transmit motion with a minimum of resistance and noise. Later when this was designed and realized it was named the involute gear. Euler was the inventor of this kind of gear and he also anticipated the underlying geometry equations now usually referred to as the Euler-Savary equations [2].

Also, the result of Euler's analysis of the motion of a rigid body around a moving axis including the effects of friction is something called Euler's disk. The technical details of motion of the Euler's disk is still being analyzed today but the key are Euler's differential equations involving the Euler angles and other [2].

Euler also found the solution to the so called Königsberg bridge problem, which gave birth to modern graph theory [1], a brilliant solution confirming once more how creative Euler was.

4. Conclusion

One can come to conclusion that Euler was a strong, creative person who had confidence in his ideas and work. One can sometimes read that Euler was not so scientific in his methods [1], in the modern sense, because he followed his intuition and played with the ideas. But that is the example how creative individuals behave.

His insight was breathtaking, his vision profound and influence significant as what is to be expected from someone so creative, as nowadays is considered that creativity is function of knowledge, curiosity, imagination and evaluation. Nothing that lacked Euler. As we could see, he didn't only wonder and came up with hypothesis, he was practical and solved many problems of his age which solutions made a foundation for science to come. Considering his publishing, he published about 50 books and more than 800 papers [1]. At present, 73 volumes of the Opera Omnia (his collected works) are in print, the publishing project that started in 1911 and still not over [1]. Euler never let the outer circumstances to affect his research, work, productivity and creativity. In the words of Andre Weil, no mathematician ever attained such a position of undisputed leadership in all branches of mathematics as Euler did for the best part of eighteenth century [1].

For the end, as creativity is also defined as the capability or act of conceiving something original or unusual, all what we can see through Euler's life and work presented here, one can follow what Laplace once said [1]: "Read Euler, read Euler. He is the master of us all."
REFERENCES