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THE LIFE AND WORK OF STEPHEN P. TIMOSHENKO

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Essay

Summary

Stephen Prokofievich Timoshenko (December 23, 1878 – May 29, 1972), was a Russian-born engineer who later emigrated to the United States. He held the chair of mechanics at the University of Michigan and later at Stanford University, and is regarded as the father of engineering mechanics in the United States. During his fruitful lifetime, Timoshenko wrote many seminal and highly influential works in the areas of engineering mechanics, such as vibration theory, elasticity, theory of plates and shells and strength of materials. Most of his works are still cited as timeless masterpieces, even by today's high academic and industry standards. The aim of this paper is to depict his extensive legacy and share with the reader a fraction of mind provoking story concerning his highly creative and innovative genius.

Key words: *Stephen P. Timoshenko; mechanical engineering; creativity; beam*

1. Introduction

In recent years, the terms creativity and innovation have become rather intertwined. One can ask thyself how to distinct them, or are they one and the same in unison? Perhaps they are different sides of the same coin? With such questions in mind, it is rather convenient to quote Paul Sloane who defines creativity as "the capability or act of conceiving something original or unusual, while innovation is the implementation or creation of something new that has realized value to others" [1]. One can draw conclusion that both terms should coexist in harmonious relationship on order for the circle of creation to complete.

Much is known of the life and influential work of Stephen Timoshenko. He has proven himself one of the brightest stars, if anyone can be dubbed as such, in the field of mechanical engineering. Most comprehensive story about his work through entire lifespan can be found in his autobiography profoundly and humbly named *As I remember* [2]. The autobiography of Timoshenko is a moving account of academic and scientific life during the most difficult years in modern history: 1900-1950. Revolutions, wars and political strife drove students, professors and scientists in Russia and Europe to great despair, and only a few like Timoshenko survived the ordeal to share their ideas and ideals with future generations. This autobiography brings to life the clash of people, cultures and ideals that shaped the course of modern engineering and science education in England, France, Germany, former Yugoslavia and, later in America. The ideas expressed, forcefully highlight absolute ideals that Timoshenko admires in scientific society. This book is full of tips for students, teachers, reviewers, educationists and the general public alike to evaluate themselves, their colleagues and their institutions. From his memories, Timoshenko recollects the great fondness of studying and explaining mathematical proofs to others, from his earliest schooldays, which is something that defined him as a future teacher from the beginning [2]. Timoshenko always stressed the great importance of mathematics as a part of mechanical engineer's education.

It may be that Timoshenko rebukes and criticizes human vanity, pride and prejudice harshly in his autobiography, but there is no denying that Timoshenko always gets it right when it comes to extracting the truth. Eternal seeker and deliverer of essence, Timoshenko is the entity that truly embodies Sloane's definitions of creativity and innovation [1], without the need to ever divide or separate them, both in the contexts of endless imagination and practical delivery. Fig. 1 denotes Stephen Timoshenko photography taken during his golden years.

Fig. 1 Stephen Timoshenko photography [6]

Vast literature, oriented non-exclusively to his body-of-work, but also his legacy as a teacher and inspirational human being, can be found [3] - [8] which serves as an additional testament of his fascinating and intriguing persona. Even highly respected mechanical-engineering authors, such as Rao [9] constantly share their admiration for Professor Timoshenko through appreciating, quoting and continuing his work. Such an appreciation can also be strongly felt in very important FAMENA textbook *Nauka o čvrstoći* by Alfirević [10], which is filled with Timoshenko references and successfully adopted lecturing methods.

1.1 Early life and education

Timoshenko was born in 1878, in the village of Shpotovka in the Konotop County of Ukraine [3]. He attended school in Romny where his schoolmate and close friend was future famous semiconductor physicist Abram Ioffe. As the son of a surveyor, he was educated as a railway engineer, which was entangled with his child memories where in his autobiography, Timoshenko recollects: "From the sand I built fortresses, castles, and, especially, railroads." [2], therefore from early childhood, his dream had been to become railway structural engineer [11]. After graduating in 1901 at the St Petersburg Institute of engineers Ways of Communication, he became teacher in this same institution 1901-1903 and then worked at the St Petersburg Polytechnic Institute 1903–1906.

In the fall of 1906 he was appointed to the Chair of Strengths of Materials at the Kiev Polytechnic Institute. The return to native Ukraine turned out to be a rather important part of his career and also greatly influenced his future personal life. As a professor at the Polytechnic Institute 1907-1911 he did research in the earlier variant of the linear-elastic Finite Element Method (FEM), the so-called *Rayleigh method*. During those years he also pioneered work on buckling, and published the first version of his probably most famous textbook *Strength of materials* [12] and [13]. In 1911 he signed a protest against Minister of Education Kasso and was fired from the Kiev Polytechnic Institute shortly after. However, in 1911 he was awarded the D. I. Zhuravski prize of the St. Petersburg Ways of Communication Institute that helped him to provide for his family financially after losing his job. He went to St Petersburg where he worked as a lecturer and then as a Professor in the Electrotechnic Institute and the St Petersburg Institute of the Railways (1911–1917). During that time he developed the theory of elasticity [14] and the theory of beam deflection which gave birth to famous *Timoshenko beam* explained in more detail in the next chapters. He also continued to further study the buckling phenomena.

After the South Russia Armed Forces of general Denikin had taken Kiev in 1919, Timoshenko moved from Kiev to Rostov-on-Don, far away from the Russian civil war of Bolshevik revolution. After travel via Novorossiysk, Crimea and Constantinople to the Kingdom of Serbs, Croats and Slovenes (*a.k.a*. the former Yugoslavia), he arrived in Zagreb, where he got professorship at the Zagreb Polytechnic Institute. In 1920, during the brief takeover of Kiev by the Polish army, Timoshenko travelled to Kiev, reunited with his family and returned with his family to Zagreb. He is remembered for delivering lectures in Russian while using as much Croatian language as he could. The students and work colleagues remember him with great fondness and were able to understand him rather well [4] and [7]. “It seems that while he believed himself to be lecturing quite successfully in the Serbian/Croatian language, his students were equally convinced that they were making great strides in learning to understand Russian…” [10].

1.2 Moving to United States

In 1922 Timoshenko moved to the United States where he worked for the Westinghouse Electric Corporation 1923 - 1927, after which he became a faculty professor at the University of Michigan where he introduced the first bachelor's and doctoral programs in engineering mechanics. He has been zealously educating both the students and all practicing engineers who are concerned with the design of machines or structures since 1927 at the University of Michigan and Stanford University, where he held increasingly important academic positions. From 1936 onward, he was a professor at Stanford University. Students remember him as “god-like figure with an aristocratic bearing; distinguished, tall and imposing - and pleasant.” [8]. During his 5 years in industry, primarily with Westinghouse, he organized the Applied Mechanics Division in the American Society of Engineers, which is now the most fruitful source of applied mechanics literature in the entire field.

In 1957 ASME established a medal named after Stephen Timoshenko and he became its first recipient. The *Timoshenko Medal* recognizes Stephen P. Timoshenko as the world-renowned authority in the field of mechanical engineering and it commemorates his contributions as an author and teacher. The Timoshenko Medal is given annually for distinguished contributions in applied mechanics.

In 1960 he moved to Wuppertal in Western Germany to spend time with his daughter. He died in 1972 and his ashes are buried in Alta Mesa Memorial Park, Palo Alto, California, next to the grave of his wife [8].

2. Scientific Work and Lecturing

Timoshenko's textbooks have been published in 36 languages. His first textbooks and papers were written in Russian. Later in his life, he published mostly in English.

He taught himself English language by reading Love's *Theory of Elasticity* and Lord Rayleigh's *Theory of Sound* [11]. Curious researcher or engineer will find an enormous joy through browsing his collected papers in one book [11] where patient reader can begin an intricate chronological scientific journey. This book is a thing of awe, worthy of the man whose collected works it holds. Papers contained in this collection contain all of Timoshenko's important scientific work, with an addition of the wonderful biographical sketch written by his close associate D. H. Young, with whom he wrote *Theory of Structures* [15] and *Advanced Dynamics* [16] textbooks.

Quick glance of the selected literature [12] - [21] also reveals Timoshenko as prolific author of many textbooks. From his firsthand experiences in industry, Timoshenko professionally addressed and solved such problems as stress concentrations, metal fatigue and buckling and stability. In his seminal work with Gere on elastic stability [17], Timoshenko developed classic solution for buckling eigen-value λ of axially loaded thin cylindrical shells. While Timoshenko's work covers a wide range in mechanics, his outstanding contributions have been mostly in the field of elasticity, both in basic theory and particularly in his masterful analysis of complicated problems to useful and practical application.

Timoshenko also dealt with resonant vibrations, which came from collaboration with his close associate J. P. Den Hartog, one of the greats in the field of dynamics and vibration. Timoshenko generally showed large professional interest in dealing with vibration problems which resulted in some of his best works in the field of dynamics [15] and [18]. It is important to note that his work on vibration problems are sometimes overshadowed by his strength [12] and [13], elasticity [14] and theory of shells [20] works, however less known fact is that his famous Timoshenko beam theory [22] (which is addressed in next chapter) actually arose from solving dynamic (vibration), not static problem. Any mathematician or any mathematically literate engineer or scientist should be obliged to own Timoshenko books. They should be used as textbooks, not so much in mathematics, as in the art of scientific communication. Those concepts should be adopted by every young scientist who aspires to express complex mathematical concepts in a manner that will help the designer of machines and structures to create a better and safer world. It can be said that Timoshenko's work blossomed with arriving to America, as he had much more time and space to develop his further theories without fearing for his, or his loved ones life, compared to previous uncertain and dangerous early days in Russia.

As from young age, Timoshenko showed grand interest in the educational methods and learned from best teachers in Russia and Europe [11], he wasn't afraid to criticize American schooling system. Moved by enthusiasm, new ideas and strong desires for active work, and as an established proficient lecturer, he observed poor mathematical knowledge of university students which yielded from adopted much lower schooling and grading criteria compared to Russian standards. Timoshenko uses term "far-cry" compared to what he was used to in his days on Russian Polytechnic Institute: "Clearly, under these conditions the training received by American engineers was *inferior* to that given engineers in Europe..." [2].

Such educational and pedagogical themes are still addressed in contemporary times and one perpetually wonders what is the best way to transfer knowledge to young generations and inspire potent and professional creativity.

3. Legacy

As important have his scientific contributions have been to the development of mechanics, they can never overshadow his human influence, as a beloved and inspiring teacher, therefore his legacy needs to be observed as a whole.

Timoshenko himself sometimes fell to frustration and self-criticism when, as he recalls "...several (papers) were read on large deflections of thin shell the mathematics of which was over my head, and I could not properly evaluate them. This plunged me into melancholy. I could no longer keep up with the rapid developments in my field. I was getting old; l was now 59…" [2]. However, sometimes one produces best results when pushed to the limits and same can be said about Timoshenko's dissatisfaction with war times and poverty in Russia and Europe which served as a propellant for his visionary work. One thing that pivotally needs to be mentioned is Timoshenko's selflessness and modesty which can be observed just by studying structure of his papers and textbooks; Timoshenko often generously cites the work of others, while consciously placing himself in the background, which is, and was rather uncommon policy in the competitive scientific world. Timoshenko spent a great deal of time preparing educational problems in mechanics, many of which were later used in his books on mechanics and strength of materials [11]. Enduring pedagogical value of his originally solved problem examples can be attributed to a long time experience as a lecturer and also long time experience as a working engineer, where as a result he managed to connect both worlds so passionately and seamlessly.

All before said implies Timoshenko's everlastingly young spirit, no matter the age, and restless creative mind always in quest for new breakthroughs. Such embodied creativity and insatiable desire for knowledge led Timoshenko to one thing that he will notably be most remembered for: *thick*, or *short* beam theory, contrary to what was exclusively used in engineering calculations until then: *long*, or *slender* Euler-Bernoulli (E-B) beam theory.

Timoshenko in 1921 presented the improved beam theory, compared to E-B beam theory, (initially) for the vibration of beams. That theory has become known as the Timoshenko, or thick beam theory [9]. If the cross-sectional dimensions are not small compared to the span of the beam *i. e.* the distance between two distinctive beam points, like a boundary condition (BC), the effects of rotary inertia and shear deformation need to be considered. The Timoshenko beam model takes into account shear deformation and rotational bending effects, making it suitable for describing the behaviour of short beams, sandwich composite beams, or beams subjected to high-frequency excitation when the wavelength approaches the thickness of the beam. Physically, taking into account the added mechanisms of deformation effectively lowers the stiffness of the beam, while the result is a larger deflection under a static load and lower predicted eigen-frequencies *i. e. natural* frequencies for a given set of boundary conditions. The latter effect is more noticeable for higher frequencies as the wavelength becomes shorter, and thus the distance between opposing shear forces decreases. If the shear modulus G of the beam material approaches infinity and thus the beam becomes rigid in shear, and if rotational inertia effects are neglected, Timoshenko beam theory converges towards ordinary (E-B) beam theory [9].

3.1 *Timoshenko beam* example

The goal of this chapter is to shed some further light on the inception of Timoshenko beam theory through an illustrative example. The problem of determining the exact value of *controversial* shear coefficient k is addressed. The fact is that even Timoshenko himself gave ambiguous results at the beginning of his research, which resulted in three different values of

k [14], [12], [22] and [23], and numerous others values in later researches conducted by scientists who continued intensively working on Timoshenko's theory [24], [25] and [26].

That implies that even the greatest of minds may stumble and fall in a trap of cognitive darkness sometimes, but they rise and shine again shortly-after, which is something that everyone who breaks the new grounds should aspire to immanently.

As a vivid simple example, Fig. 2 contains the schematics of a common experimental setup which represents three point bending test (3PBT) in which rather short *i. e.* Timoshenko thick beam is loaded in a way which denotes analytical model of a simply supported beam.

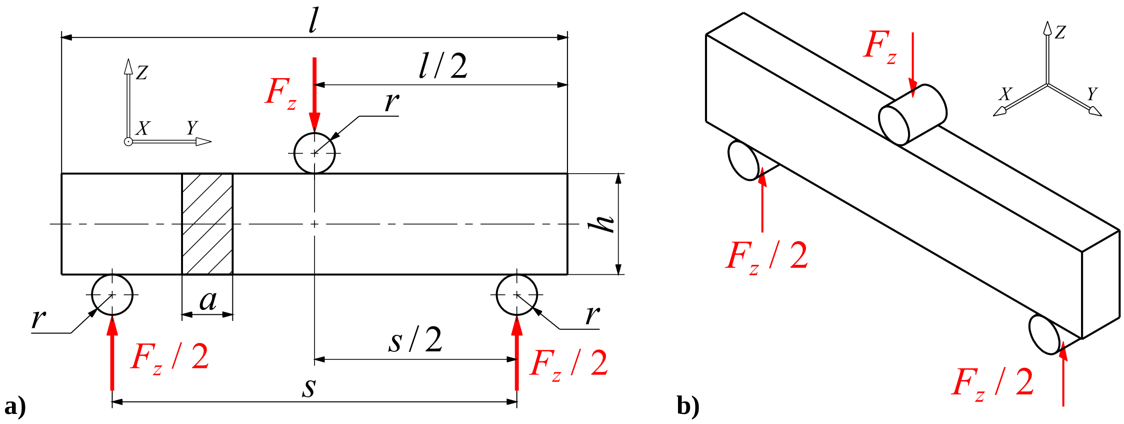

Fig. 2 Timoshenko beam in three point bending test: **a)** front view, **b)** isometric view

According to *e. g.* Öchsner [24], well known explicit closed form solution for maximum static deflection $w_{z,\max}$ of simply supported Timoshenko beam is

$$w_{\Sigma} = w_{z,\max} = \left(\frac{F_z s^3}{48EI_x}\right)_{\text{Euler-Bernoulli}} + \left(\frac{F_z s}{4kA_0 G}\right)_{\text{Timoshenko}}, \tag{1}$$

where F_z is the upper roller static force, s is the range in between two lower rollers, E is the Young modulus, $I_x = a \times h^3/12$ is the axial inertia moment with regards to x axis, $A_0 = a \times h$ is the cross section area, G is the shear modulus and k is the debatable and controversial Timoshenko shear coefficient. Full length of beam is denoted by l, and r is the radius of rollers, which is trivial with regards to analytical computation. Index "Σ" in w_{Σ} denotes cumulative influence of both Euler-Bernoulli and Timoshenko separate parts. It is obvious that for long and slender beam where $s^3 \gg s$, right Timoshenko term becomes negligible compared to left term and equation (1) denotes Euler-Bernoulli beam exclusively. However, for shorter, *thick* beams where *slenderness* ratio h/a becomes significantly large, such approximation *i. e.* using long, *slender* beam definition is no longer valid. Rule of thumb is that slenderness ratio h/a should be smaller than *e. g.* 1/10…1/20, in order to obtain valid and sufficiently correct *engineering* results where relative error[1] assumes values $E_{\text{rel}} \leq 5$ %, which was parametrically evaluated in [24]. Such low ratio can sometimes be overly conservative, compared to needed computational effort. Alfirević [10] claims that slenderness ratio should $h/a \leq 1/4…1/5$ for relation $E_{\text{rel}} \leq 5$ % to hold, which can be too liberal and lead to erroneous results for different boundary conditions. As an interesting fact, commercial software package based on FEM, *Simulia Abaqus* documentation recommended slenderness ratio $h/a \leq 1/15$ 0 can be found. All stated above implies that firm division between E-B and Timoshenko beam cannot be so easily distinguished.

[1] Relative error E_{rel} is defined by expression $E_{\text{rel}} = (w_{z,\text{Timoshenko}}/w_{z,\text{E-B}} - 1) \times 100$ (%).

Timoshenko (1921) [22], in his first paper, used the value of k = 2/3 ≈ 0.667 for a rectangular section. In derivation process, he considered the effect of shear on the deflection of a beam to be equal to the shear angle at the *centroid*. It is interesting to note that research was conducted and paper was written in former *Yugoslavia* in 1920. Identical result for k, although implicitly, can be found in his famous *Theory of elasticity* book with Goodier [14]. In *Strength of Materials* [12], Timoshenko gives an explicit result for k, although in inverse form, where his notation is $\alpha = (1/k =)\ 3/2$. In 1922 [23], Timoshenko set k = 8/9 ≈ 0.889 in order to bring the prediction of his frequency equation for the supported-supported end condition into closer agreement with the two-dimensional theory based on the assumption of plane stress. It is most interesting to note that, although implicitly, in this paper Timoshenko also gives the value of shear coefficient as $k = 5(1+\nu)/(6+5\nu)$ [26], where ν is the Poisson's ratio. That raises further dubious question whether the k coefficient can be obtained unanimously and explicitly in closed form solution. As of further interesting note, as a continuation of previous work, this paper was written in Zagreb 1921.

Table 1 shows the values of shear coefficient k value interesting historical evolution for beam rectangular cross section. Three successive Timoshenko values/expressions [22] and [23] and one Cowper [25] expression are presented as follows.

Table 1 Shear coefficient k for beam rectangular cross section $A_0 = b \times h$, history and evolution

	Timoshenko (1921.)	Timoshenko (1922.)	Timoshenko (1922.)	Cowper (1966.)
k	$\frac{2}{3}$	$\frac{8}{9}$	$\frac{5(1+\nu)}{6+5\nu}$	$\frac{10(1+\nu)}{12+11\nu}$

More than 40 years later, Cowper [24] was one of the many authors that further studied and upgraded Timoshenko results from his previous two papers [22] and [23]. As already shown in Table 1, Timoshenko and Cowper shear coefficients k are therefore respectively

$$k_{\text{Timoshenko}} = \frac{5(1+\nu)}{6+5\nu}, \quad k_{\text{Cowper}} = \frac{10(1+\nu)}{12+11\nu}. \tag{2a,b}$$

Both expressions are still used in literature extensively[2] [24]. For instance, *Simulia Abaqus* documentation 0 quotes Cowper's paper [25] and notes that expression (2b) is implemented, *i. e. hardcoded* in its numerical solver. It is also interesting to note that if Poisson's ratio ν is set to zero, Timoshenko and Cowper expressions (2a) and (2b) both give same solutions, *i. e.* $k_{\text{Timoshenko,Cowper}}$ = 5/6 ≈ 0.833, which can also be found in paper by Kaneko [26], where author gives more detailed insight of various k results and expressions obtained from numerous other authors. Additional literature can be found on the topic [24].

Sole purpose of this short theoretical review and analysis was to show aspiring scientist's inner turmoil and dubious search for truth, even if it means revising and restructuring everything that has been done before, even by thyself. Most importantly, that includes objective self-critique and uncompromising will to progress further in eternal quest for true knowledge and creativity.

Exact and explicit determination of shear coefficient k is still a hot and attractive topic, and is solved analytically, numerically and experimentally, even today after almost 100 years since its first inception.

[2] Author of this text performed independent analytical and numerical research where it was found that both expressions for shear coefficient k are sufficiently correct for engineering purposes.

4. Conclusion

Professor Timoshenko was undoubtedly one of the *giants* of 20_{th} century theoretical and applied mechanics. He has enriched the lives of thousands of his students and colleagues during his many years of active work. His *magnum opus* consists of many addressed and successfully solved problems in mechanical engineering and includes both numerous books and scientific papers. One can find obvious link between his life and his work, where dynamics and turbulence of his unfortunate time circumstances (most notably a necessity to flee from homeland due to a political turmoil at the time), later resulted in his most inspired and valuable research. Timoshenko was always in motion, always on the brink of opening and broadening new horizons. One can say that he was indeed a true free spirit and limitless soul, punished by destiny to always keep him on the move, but forever touched by creative divinity, all of which forged him into a remarkable person that he was.

While others sometimes delighted and dwelled in pure mathematical abstractions, Timoshenko did not scorn a concrete result or even a simple numerical answer. He always offered the bridge between pure mathematics and practical design, which we desperately need. Through him and his able colleagues and associates, we have progressed from "mechanics" through "applied mechanics" and finally to "engineering mechanics" of which Timoshenko is the great disciple. There is substantially no repetition in Timoshenko's works, and nothing he wrote is less than important, if not stellar. However, one main and constant theme of his work is the avoidance, wherever possible, of a complicated exact analysis by a simple and sufficient approximation. In current times, when academy and industry seem to drift further away, one should immensely cherish such approach and try to remember the reasons why the science and philosophy emerged in the first place – to solve problems, not to widen the gap.

Timoshenko's work included countless novel comprehensions regarding physical understanding of the world which surrounds us, and one asks thyself how was such creative workflow possible back then, without modern benefits of internet shared knowledge and fast computers. The wondrous legacy of Timoshenko will be inexhaustible inspiration for future generations and prove of immanent stimulating interest to young engineers and scientists of curious mind with thirst and hunger for knowledge, all across the globe.

Author of this paper shares his deepest admiration and respect for Professor Timoshenko and hopes that he has managed to bring closer topic of life and legacy of this exquisite human being to respectable reader.

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